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Floyd K. Richtmyer

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FLOYD KARKER RICHTMYER, Editor of the *Journal of the Optical Society of America* since September 1, 1932, died suddenly at his home in Ithaca November 7, 1939. In recording his passing the Journal and the Society pay tribute to one whose unselfish and untiring service has been an essential element in their creation and in the maintenance of their existence during these troubled times.

Dr. Richtmyer was a charter member of the Society, and contributed the leading article "Opportunities for Research" in the first number of the Journal. In spite of the diversity which his activities later attained and the multitude of honors and responsibilities which came to him, he somehow always found time to devote to the welfare of the Society. In one capacity or another he was a member of its Executive Council, and later the Board of Directors, almost continuously from 1918 until his death. He was Vice President in 1918–19, President in 1920, Associate Editor of the Journal from 1917 to 1921, and served as Acting Secretary from 1924 to 1926. He was the principal agent in obtaining the support which made possible the enlargement of the Society's activities by establishment of the combined *Journal of the Optical Society and Review of Scientific Instruments*, and as Assistant Editor-in-Chief and Business Manager of that periodical from 1922 to 1930 he played an indispensable part in its development.

When *The Review of Scientific Instruments* was set up as a separate periodical in 1930, he continued as Assistant Editor-in-Chief of both journals. In 1932, when Paul D. Foote felt compelled to relinquish the editorship, the Society naturally turned to Richtmyer as the man best qualified to continue the work. The burden of administrative duties which he already bore as Dean of the Graduate School at Cornell, Chairman of the Division of Physical Sciences of the National Research Council, and active member of a long list of committees of various kinds might well have deterred him from taking on added responsibility. Nevertheless his interest in the success of the Journal and the Review was so deep that he cheerfully accepted the call to take charge of them. Those who have been associated with him in the work know with what thoroughness and conscientious care he performed the arduous duties of Editor. One may well repeat for him the tribute which he paid to Dr. Foote in 1932 in the following words: "To have been thus instrumental in starting and in conducting for a long period of years two important scientific journals is an achievement of which anyone may be proud. On behalf of his colleagues on the editoral board, and of the many readers of the *Journal of the Optical Society of America* and of *The Review of Scientific Instruments* we take this occasion to express our sincere appreciation of his untiring energy and of his

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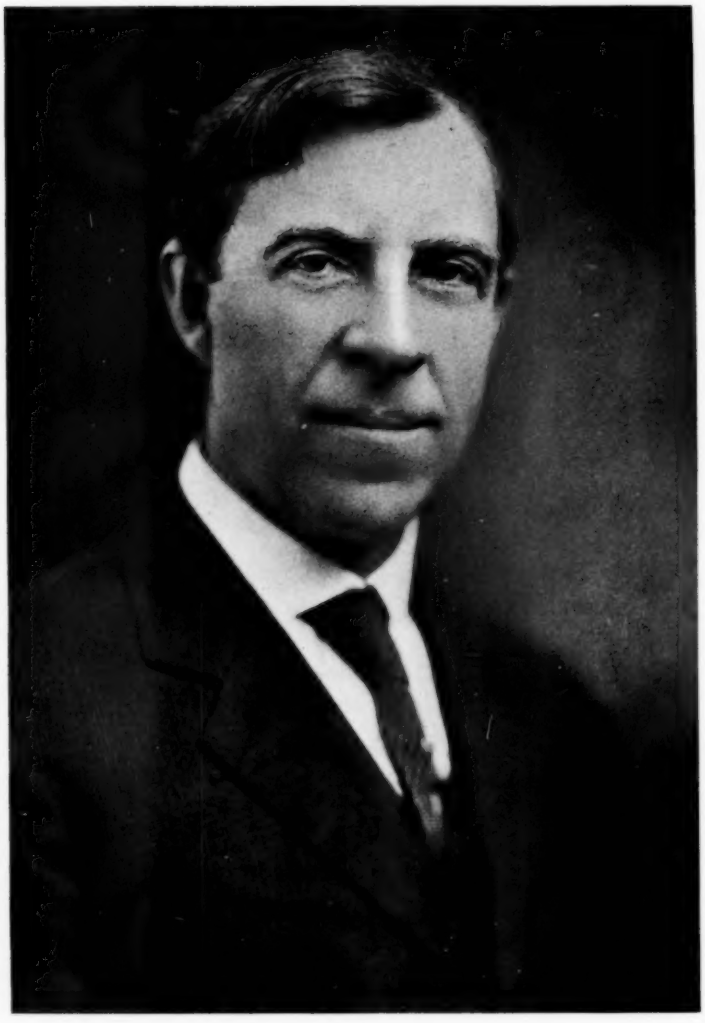

Floyd K. Richtmyer

devotion to the editorial work which he has so ably done."

But with all his devotion to the Optical Society and his outstanding record of service to it, his views and his aims were never bounded by selfish or limited interests. He saw that Society not as an organism set apart from others or competing with them, but rather as one member of a greater fellowship representing all branches of physics and working together with all agencies of science for the good of mankind. So it was natural for him to help in bringing about the collaboration of societies which resulted in the formation of the American Institute of Physics. On its Governing Board he was a representative of the Optical Society from 1931 to 1937. His effective aid in strengthening the Institute and in promoting effective working relations between it and the constituent societies was by no means the least of his contributions to the development of science in America and incidentally to the Optical Society itself. The extent of his influence is well illustrated by his unique record of having held the presidency of three of the five societies

which make up the Institute; he was President of the American Physical Society in 1936 and of the American Association of Physics Teachers in 1937–8, and to him such positions meant not merely honors, but opportunities for serious work in behalf of the societies.

That the breadth of his views and the fairness of his judgments were appreciated by his fellow workers is evidenced by his repeated selection as chairman of the Division of Physical Sciences of the National Research Council (1930–1935) and as a member of the Fellowship Board of the Council (1930–1937). He was actively interested in the Society of Sigma Xi and was president of the national society from 1924 to 1926.

Too often administrative responsibilities and extraneous activities have diverted promising workers from research. Such a result seemed inevitable in Dr. Richtmyer's case, but along with all his other occupations he continued to carry on personal scientific work and to direct the researches of a group of graduate students. When one considers the amount of time and energy which he gave to other duties, the number of significant scientific papers which he produced is astonishing.

His doctoral dissertation "On the photoelectric effect with alkali metals" included a study of the relation between photoelectric current and intensity of illumination. This work was done at a time when there was lively interest in photometry and illumination, and Richtmyer worked under the direction of Professor Edward L. Nichols, who had been one of the first in this country to make a scientific study of photometric problems. Quite naturally therefore his attention was directed to this field, and for a decade most of his publications were on photoelectric phenomena and measurement of light. These papers were distinguished by careful measurements and painstaking analysis of data rather than any particularly novel ideas, but they established the reputation of the author as a thorough investigator with a keen sense of the practical applications of his work.

In 1918 Dr. Richtmyer was advanced to a full professorship at Cornell, and within the next year began the program of research on x-rays which has given him and his students an outstanding reputation. His capacity for careful measurement was first applied to the establishment of the laws of absorption of x-rays and to the search for the supposed "*J*" radiation. In 1927 he turned to the study of satellite lines and their relation to atomic structure and ionization. For the last decade his administrative duties, and especially the large amount of traveling which they required, prevented his personal participation in systematic laboratory work; but he continued to keep in close touch with his group of graduate students who responded to his leadership by producing a series of important papers on the problems growing out of the study of satellites which he began. His own creative work and that which he inspired in others doubly justified his election to the National Academy of Sciences, of which he became a member in 1932.

He was an outstanding teacher, partly because of his insistence on thorough treatment of fundamentals and on the necessity of having quantitative as well as qualitative knowledge. This attitude and his belief in the essential unity of physics, ancient and modern, are most explicitly emphasized in his textbook, *Introduction to Modern Physics*.

Physically and mentally Dr. Richtmyer was solid and substantial, a rock on which to build. Idealistic but not visionary, he made far-reaching plans, but his head was never so far in the clouds that his feet failed to rest on solid ground. So in his scientific work he was an excellent liaison between the old physics and the new. Receptive to the newer developments and appreciating their importance, he still believed that careful measurement was the real foundation of knowledge and that it was worth while to strive for one more decimal place.

It is not often that one finds such a combination of thorough dependability, painstaking accuracy, sound judgment and tireless industry as he had. Yet after all it is not for those qualities that we shall miss him most. We shall cherish even more the memory of the steadfast courage with which he met doubts and difficulties, and the warm human sympathy which welled so freely from the depths of his robust spirit.

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The Limiting Sensitivity of the Alternating-Current Method of Photo-Cell-Current Amplification

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The limiting sensitivity of an alternating-current system of amplification is considered. The practical limit is determined theoretically and experimentally for measured circuit parameters. A particular circuit using an electrometer-tube and a cesium-antimony cell is shown to be capable of detecting 4.5×10^{-15} watt in 30 seconds. Data on various input-tubes and photo-electric cells are given.

THE most general technique for measuring small photo-cell currents is the use of a direct-current amplifier. In certain cases, and also in order to approach the limit of sensitivity with maximum stability, it is advantageous to interrupt the light to be measured with an optical shutter and to measure the pulsating light with an alternating-current amplifier. The following analysis is a general review of the theoretical sensitivities possible.[1-4]

It has been shown[5] that the input noise level of a circuit consisting of a resistance R shunted by a capacity C, through which a thermionic shot current I_g is flowing, and which is connected to the input of an amplifier whose tube noise is given by the equivalent noise resistance R_t, is given by

$$E_n^2=4KT\{(1/2\pi C)(\tan^{-1}[2\pi RC(f_2-f_1)/(1+4\pi^2R^2C^2f_1f_2)])\times(1+19.4\,I_gR)+R_t(f_2-f_1),\quad(1)$$

where the amplifier is supposed to have uniform amplification between the frequencies f_1 and f_2 and zero elsewhere, and where K = Boltzmann's constant, T = absolute temperature, C = sum of static and dynamic input-capacities. The circuit of input is shown in Fig. 1 where $I_g=I_t+I_p$, I_t = grid-current from input-tube, I_p = dark current of photo-cell. C is given by $C=C_p+C_{gf}+(1+\mu_e)C_{gp}+C_s$ where C_p = photo-tube capacity, C_{gf} = grid-to-filament capacity of input-tube, C_{pg} = grid-to-plate capacity of input-tube, C_s = stray capacity to ground, μ_e = effective amplification of input stage.

The signal-voltage squared due to the root-mean-square signal-current I_s flowing through the photo-cell is given by

$$E_s^2=I_s^2R^2/(1+4\pi^2f_o^2R^2C^2).\quad(2)$$

The condition that the signal voltage be a maximum is that $2\pi f_oRC\gg1$ and this same condition makes E_n small provided $19.4\,I_gR$ is not too great compared to unity and that

$$(f_2-f_1)/(f_1f_2)^{\frac{1}{2}}\lesssim1,$$

that is, that the percentage band width is comparatively small. Under these circumstances (1) and (2) reduce to

$$\overline{E_n^2}=4KT(f_2-f_1)\times[R_t+(1+19.4\,I_gR)/RC^24\pi^2f_1f_2],\quad(3)$$

$$E_s^2=I_s^2/C^24\pi^2f_o^2.\quad(4)$$

The ratio of signal to noise is then given by

$$E_s^2/\overline{E_n^2}=I_s^2R/4KT(f_2-f_1)\times[1+R(19.4\,I_g+R_tC^24\pi^2f_o^2)],\quad(5)$$

where the modulation frequency f_o is taken so that $f_o^2=f_1f_2$. This will always be approximately true if $(f_2-f_1)/(f_1f_2)^{\frac{1}{2}}<1$. f_o is then the geometrical mean frequency of the amplifier band width. The maximum value of this ratio is obtained when $R(19.4\,I_g+R_tC^24\pi^2f_o^2)\gg1$ and is given by

$$E_s^2/\overline{E_n^2}=I_s^2/4KT(f_2-f_1)(19.4\,I_g+R_tC^24\pi^2f_o^2).\quad(6)$$

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** Now at the University of Rochester, Rochester, New York.
[1] J. B. Johnson and F. B. Llewellyn, Elec. Eng. **53**, 1449–1453 (1934).
[2] B. A. Kingsbury, Phys. Rev. **38**, 1458–1476 (1931).
[3] E. Steinke, Zeits. f. Physik **38**, 378–403 (1926).
[4] F. V. Orbán, Zeits. f. tech. Physik **14**, 137–143 (1933).
[5] E. A. Johnson and A. G. Johnson, Phys. Rev. **50**, 170 (1936).

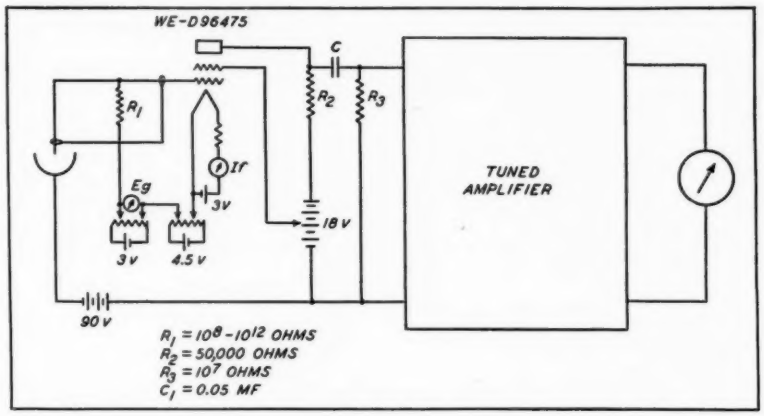

FIG. 1. Circuit-diagram of photo-cell amplifier.

For practical reasons, however, the constants must sometimes be chosen such that $R(19.4\,I_g + R_t C^2 4\pi^2 f_o^2) \ll 1$, as will be brought out in the following discussion, and in this case we have simply

$$E_s^2/\overline{E_n^2} = I_s^2 R/4KT(f_2 - f_1). \qquad (7)$$

Whether (6) or (7) is applied to any particular case, it is always true that maximum sensitivity is obtained when the zero signal current I_g, the equivalent tube noise resistance R_t, and the circuit capacity C, are all a minimum. The factors R and C are further restricted by the fact that they determine the time constant of the circuit, since the coefficient of growth of the voltage across the grid is given by $[1-e^{-t/RC}]$ where t is the time measured from the instant that the pulsating light is applied to the photo-cell. The value of C that can be attained varies in a practical case from 5×10^{-12} to 20×10^{-12} farad, depending on the input-tube, photo-cell, and wiring. Using a mean value of $C=10\times10^{-12}$ farad the value of R required for the signal to reach 95 percent of its steady-state value in time T, the "period" of the circuit, is given by

$$T=3R\times10^{-11}.$$

For a "period" of 30 seconds $R=10^{12}$ ohms and if it is possible to tolerate a still longer period, say 300 seconds, R must still be not greater than 10^{13} ohms. For periods of 0.3 to 300 seconds R lies between 10^{10} to 10^{13} ohms.

The values of I_g and R_t are not independent quantities since the value of I_g depends partly on the grid current of the input-tube, while R_t is also a parameter of the input-tube. In general it is found empirically that small values of R_t are associated with large values of grid current. In addition small values of R_t are usually associated with large values of dynamic tube capacity.

The values of R_t, C, I_g, of several tubes are given in Table I together with their operating potentials and filament current. The values of R_t are larger than at the normal operating conditions but the values of I_g are very much lower.

There are wide variations in the parameters of any particular type of tube. The values of R_t given in Table I in general apply at frequencies above 100 c.p.s. the value of R_t given for the RCA 38 and electrometer tubes is good down to ten c.p.s. At low frequencies R_t becomes a function of frequency and can no longer be assumed constant. For most tubes R_t increases inversely as the frequency below some critical frequency, which is very different for the various types of tubes. For this reason care must be taken in applying the analysis considered here unless the variation in R_t with frequency is taken into account.

It can be shown that the smallest signal that can be detected in the presence of a statistical voltage E_n depends on the band width and the period of the output-meter. The smallest detectable signal is given by

$$E_s=(5\lambda/(f_2-f_1))^{\frac{1}{2}}E_n \qquad (8)$$

TABLE I. *Values of tube parameters for various tubes.*

TUBE	C FARADS $\times 10^{12}$	R_t OHMS $\times 10^{-4}$	I_g AMPERES $\times 10^{13}$	I_f AMPERES	E_{c1} VOLTS	E_{c2} VOLTS	E_p VOLTS
RCA 38	24	3.8	2.5	0.28	−1.8	+ 6	+20
DE V	6.6	7.0	2.4	[E_f=2.35]	−2.0	....	+20
WE 259B	7.2	3.6	5.0	1.3	−1.5	+17	+19
262A	17	2.6	18	0.28	−1.8	....	+17
379A	6.1	10	0.6	0.28	−2.2	+ 9	+25
Electrometer tubes Approximate values	3.0	40	0.01	....	....	....	....

to a certainty of one part in 200, where λ is the decay constant of the output-meter, that is, where the meter-reading decays as $e^{-\lambda t}$. The smallest voltage that can be detected for a given band width and input circuit thus depends upon the period of the meter. Since the time constant of the input circuit also determines the time required to make a reading, for maximum sensitivity $1/RC$ and λ should be of approximately the same value. Eqs. (6) and (7) thus become

$$I^2_{o\,\min}=20KT\lambda(19.4\ I_g+R_tC^2 4\pi^2 f_o^2), \quad (9)$$

$$I^2_{o\,\min}=20KT\lambda/R. \quad (10)$$

If the photo-cell sensitivity is A amperes per watt, and if the unmodulated light intensity is L watts, then the mean-square fundamental signal current I_o^2 is given by

$$I_o^2=A^2L^2/8, \quad (11)$$

where it is supposed the photo-cell-sensitivity is independent of frequency.

Equations (5) to (11) then become for the minimum detectable amount of light

$$\Delta L=8.1\times10^{-10}[1+R(19.4\ I_g+R_tC^2 4\pi^2 f_o^2)]^{\frac{1}{2}}[\lambda/RA^2]^{\frac{1}{4}} \quad (12a)$$

$$=8.1\times10^{-10}[19.4\ I_g+R_tC^2 4\pi^2 f_o^2]^{\frac{1}{2}}[\lambda/A^2]^{\frac{1}{4}} \quad (12b)$$
$$1\ll R[19.4\ I_g+R_tC^2 4\pi^2 f_o^2]$$

$$=8.1\times10^{-10}[\lambda/RA^2]^{\frac{1}{4}} \quad (12c)$$
$$1\gg R[19.4\ I_g+R_tC^2 4\pi^2 f_o^2].$$

The cases that are of special interest are where (*I*) the sensitivity is limited by the dark current of the photo-cell or by background light and (*II*) where the sensitivity is limited only by the allowed period of observation, that is, Eq. (12*c*) applies. For case (*I*) if I_g is due to a steady background light L_o we have

$$\Delta L=3.6\times10^{-9}[\lambda L_o/A]^{\frac{1}{2}}. \quad (13)$$

For case (*II*) if we let $1/RC=\lambda$, that is, if the time constant of the input circuit and of the meter are made approximately equal we have

$$\Delta L=8.1\times10^{-10}C^{\frac{1}{2}}\lambda/A.$$

The limiting sensitivity that may be obtained for case (*II*) using a photo-cell sensitivity of 20×10^{-4} ampere per watt [a cesium-oxide cell cooled with dry ice], an input capacity of 10^{-11} farad, and band width of 10 c.p.s. at 100 c.p.s. is given in Table II. The sensitivities calculated for Table II have been verified experimentally.

In all cases in order to measure the light intensity to ±2 percent approximately 25 times as great intensity is required as the minimum detectable intensity.

The choice of tubes obviously depends upon the constants of the input circuit and the modulation frequency. The factor

$$B=10^{-4}/[19.4\ I_g+R_tC^2 4\pi^2 f_o^2]^{\frac{1}{2}}$$

may be taken as a factor of merit of a tube and is given in Table III for various frequencies. The case of electrometer tubes is unusual since in their case R_t has no particular significance and cannot be conveniently measured. This is due to the fact that the amplification of an electrometer tube is in general less than unity so that its plate

TABLE II. *Limiting sensitivity of light detection.*

λ SEC.$^{-1}$	"PERIOD" SEC.	ΔL WATTS $\times 10^{13}$	R OHMS $\times 10^{11}$
1.0	3	13	1
0.3	10	3.8	3.3
0.1	30	1.3	10
0.03	100	0.38	33
0.01	300	0.13	100

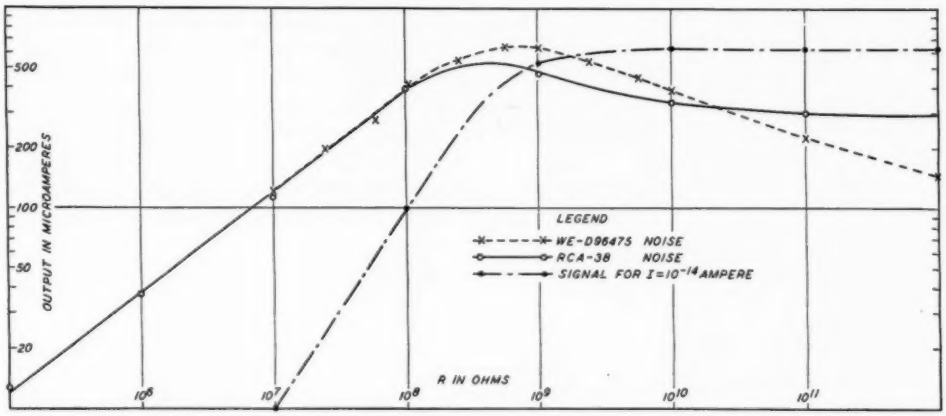

FIG. 2. Variation of amplifier output-current *versus* input resistance for noise level and for a signal current of 10^{-14} ampere.

circuit and the second stage of the amplifier determines the fundamental noise level. The value of R_t given for electrometer tubes is based upon a voltage amplification of 0.5 in the first stage, a combined first-stage plate-circuit noise plus second-stage noise of 100,000 ohms, while the first-tube noise is neglected. This approximates well the experimental electrometer tube behavior in the type of circuit proposed in this analysis.

From Table III it can be seen that at frequencies which can ordinarily be used in modulating the light-beam, the factor of merit is determined principally by the grid current. Furthermore, since Table II can be used only if $(10^5B^2/R)\gg 1$, it can be seen that this condition is approximately fulfilled only for electrometer tubes for input resistances up to 10^{12} ohms. In general the use of filament-type tubes, such as electrometer tubes and the DEV in alternating-current amplifiers, is sometimes objectionable because of their high microphonic level and should be resorted to only if extreme sensitivity is required. For general use a tube such as the WE 379A or some other cathode type of tube is preferable.

TABLE III. *Factor of merit B of various tubes.*

TUBE	f_0 (CYCLES PER SECOND)		
	10	100	1000
RCA 238	14	14	7.7
DEV	14	13	5.0
WE 259B	10	9.9	6.1
WE 262A	5.3	5.3	3.5
WE 379A	29	25	4.9
Electrometer tubes	190	35	3.5

EXPERIMENTAL RESULTS

The limiting sensitivity has been verified experimentally for the RCA 38 and for the WE D 96475 electrometer tube, using various photoelectric cells. The circuit diagram of the measuring circuit is shown in Fig. 1. The amplifier was tuned to ten cycles per second and has been described elsewhere.[6] Fig. 2 shows the experimental variation of tube plus circuit noise output-current as a function of the input-resistance, and the corresponding signal current for a fixed signal. These curves illustrate the advantage to be gained by using a very high input-resistance since it can be seen that the noise increases more slowly than the signal and finally again decreases.

The limiting sensitivity was determined by measuring the direct current flowing in the photo-cell and then flickering the light from the same source maintaining the geometry and determining from the noise and the alternating-current signal the minimum detectable current. Using a Kunz potassium-hydride cell and the WD D96475 as an input-tube the observed and calculated sensitivities are given in Table IV using the measured constant $\lambda=0.5$, $R_t=4\times10^5$ ohms, $C=25\mu\mu$f, $I_0=2\times10^{-13}$ amp. (photo-cell dark-current), $f_0=10$ c.p.s.

[6] E. A. Johnson, Rev. Sci. Inst. **9**, 263–266 (1938).

TABLE IV. *Current-sensitivity of photo-cell amplifier-circuit.*

GRID RESISTANCE IN OHMS	MINIMUM DETECTABLE CURRENT IM AMPERES	
	CALCULATED $\times 10^{15}$	MEASURED $\times 10^{15}$
10^{8}	58	63
10^{9}	18	20
10^{10}	5.7	6.2
10^{11}	2.2	2.4
10^{12}	1.4	1.5

With care the photo-cell dark current of the best photo-cells can be made as low as 1×10^{-15} ampere and the grid current of an electrometer tube can be made about as small. It is also possible to make the input capacity as low as ten mmf. It is then of interest to calculate an extreme case where it is assumed that maximum sensitivity is desired with an allowable period of 30 seconds. Then using the constants $\lambda=0.04$ sec.$^{-1}$, $R_t=4\times10^{5}$ ohms, $C=10^{-11}$ farad, $I_g=2\times10^{-15}$ ampere, $f_o=5$ c.p.s., and $R=3\times10^{12}$ ohms we obtain from (12*a*) $\Delta LA=\Delta I_s=9\times10^{-17}$ ampere.

This value is the modulated *direct current that can be detected.* A root-mean-square alternating current of 3.2×10^{-17} ampere can be detected. Again, to assume an extreme case, if we take a photoelectric sensitivity of 20 milliamperes per watt (a Cs−Sb cell at 2400A) it would be possible to detect a modulated direct light intensity of 4.5×10^{-15} watt.

Although it is theoretically possible to detect even smaller signal currents by decreasing R_t, C, λ, I_g, and f_o, and by increasing R, in practice no appreciable gain can be obtained with the thermionic amplifier tubes now available and without undesirably increasing the period of the detecting system.

In the preceding analysis it has been assumed that no extra noise is developed in the photoelectric cell except that due to the shot-effect of the photo-cell current. In cells using gas-amplification, an abnormal shot-effect[2] occurs which tends to make the use of gas-filled cells less favorable, even though their sensitivity is higher, that is, the increase in noise may be greater than the increase in signal. There is, however, another source of noise in certain types of photoelectric cells. This noise is the "granular" noise investigated by Pearson[7] and it occurs in all cells in which the distilled film of the photosensitive surface provides a leakage path between anode and cathode. If such leakage is appreciable, and this will occur when measurements at the limit are considered for leakage resistances of 10^{12} to 10^{13} ohms or less, the granular noise may exceed the other limiting noise by several powers of ten. Unless great care is taken in their construction, most cells which contain free cesium or sodium show the presence of this noise. The granular noise disappears if the ohmic leakage of the cell is reduced by "torching" the cell to drive the cesium or sodium from the leakage path.

A number of commercial photoelectric cells of various types have been tested, as well as several special cells loaned to us by the Bell Telephone Laboratories. The results of these tests are given in Table V and give some indication of the limitations of various cells.

[7] G. L. Pearson and C. J. Christensen, Bell Sys. Tech. J. **15**, 197–223 (1936).

TABLE V. *Dark current and granular noise of various photoelectric cells.*

TYPE	MANUFACTURER	DARK CURRENT	GRANULAR NOISE
Cs-O	RCA 922	1.5×10^{-9}	No
Cs-O	GE PJ23	1.6×10^{-10}	No
Cs-O	NBS (experimental)	4×10^{-9}	No
Cs-O	Bell Telephone Laboratories (double-ended experimental)	1.2×10^{-11}	No
Na	Burt	7×10^{-12}	Yes
Na	Experimental	9×10^{-13}	Slight
Cs-Mg	Westinghouse (production-type)	1×10^{-8} to 1×10^{-10}	Yes
Cs-Mg	Westinghouse (experimental)	6×10^{-13}	Yes
Potassium hydride	Kunz	6×10^{-13}	No
Cs-Sb	Experimental	6×10^{-10}	Yes
Cs-Sb	Double-ended experimental	5×10^{-13}	Slight
K-O	Bell Telephone Laboratories (experimental)	1×10^{-10}	?

Although there are other photoelectric detectors, such as the electron multiplier and the light counter, the stability and reliability of a photoelectric cell and its associated apparatus often makes its choice preferable. In any case the limitation of measurement will be the same for any photoelectric device in so far as dark current is concerned, that is, the shot-effect of the dark current will produce a statistical fluctuation in the output-meter which will set the same lower limit of measurement. The electron multiplier and light counter are not subject to thermal agitation and the effect of input resistance on the response period, but they are subject to other limitations.

We are indebted to A. E. Whitford for discussion of this paper and to R. C. Meyer for his accurate measurements of tube parameters.

The Measurement of Light Scattered by the Upper Atmosphere from a Search-Light Beam

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Rayleigh scattering from atmospheric gases in the path of an intense search-light beam has been measured up to heights of 34 km. It is found that above a limit of haze, which varies on different days from four to ten km, the scattering agrees with that calculated upon the assumption of the usual atmosphere. Clouds have been occasionally observed at heights of 12 km. It is shown that the method can be extended to allow measurements up to heights of 70 to 90 km and to determine the distribution of ozone at low altitudes, as well as the study of other atmospheric problems.

INTRODUCTION

THERE are many physical problems of the upper atmosphere in regions which are inaccessible by ordinary methods of measurement. Such problems, upon which only a scanty attack has been made, are the variation of density, of composition, and of temperature with height up to great heights. The problem of composition—for example, the distribution of ozone below 25 km, the presence of water-vapor between 10 and 20 km—is of particular importance to meteorologists. In the hope of developing a technique for the study of the region above 30 km, the idea of utilizing scattered light from the path of a search-light beam as a measure of density *versus* height has been proposed several times,[1, 2] and measurements have been made by E. O. Hulburt using a photographic exposure.[3]

Hulburt's method registers the average intensity over the period required for the photographic exposure; it is not suited to an evaluation of the presence of varying absorption or scattering such as that due to the movements of incipient of water-vapor. Using this photographic technique the ultimate limit is set by the maximum photographic gamma attainable and the contrast between the intensity of the beam and that of the night sky.

We have developed the method discussed by Tuve, Johnson and Wulf[2] of using a modulated search-light beam, in conjunction with a photoelectric cell and tuned alternating-current amplifier at the receiver. The limiting height in this case is also set by the background fluctuation introduced by the light from the night sky. Measurements have been made on an absolute basis and the good agreement with theory allows the extension of the method with improved apparatus to considerably greater heights than those reported here.

THEORY

The geometry used in our experiments is shown in Fig. 1. In order to compare the experimental results with the calculated ones, and to interpret the receiver-response in terms of the variation of density with height, it is necessary to determine the radiant energy in the beam, the amount and direction of scattering, and the spectral response of the receiver to the scattered light. The second column of Table I gives the energy in watts for wave-length bands 400A wide in a search-light beam using a 110-degree, 24-inch aluminized mirror, and 11-mm high intensity motion-picture projection carbons; the data for the test were obtained from a graph supplied by the National Carbon Company. The calculations assume that all the light in the useful beam comes from the positive crater and is reflected 100 percent for all wave-lengths from the mirror. The actual reflection from aluminum averages 85 percent.

From Rayleigh's theory of molecular scattering, the amount of light scattered from a

* Now at the University of Rochester, Rochester, New York.
** Now at the Massachusetts Institute of Technology, Cambridge, Massachusetts.
[1] E. H. Synge, Phil. Mag. **9**, 1014–1020 (1930).
[2] M. A Tuve, E A. Johnson and O. R. Wulf, Terr. Mag. **40**, 452–454 (1935).
[3] E. O. Hulburt, J. Opt. Soc. Am. **27**, 377–382 (1937).

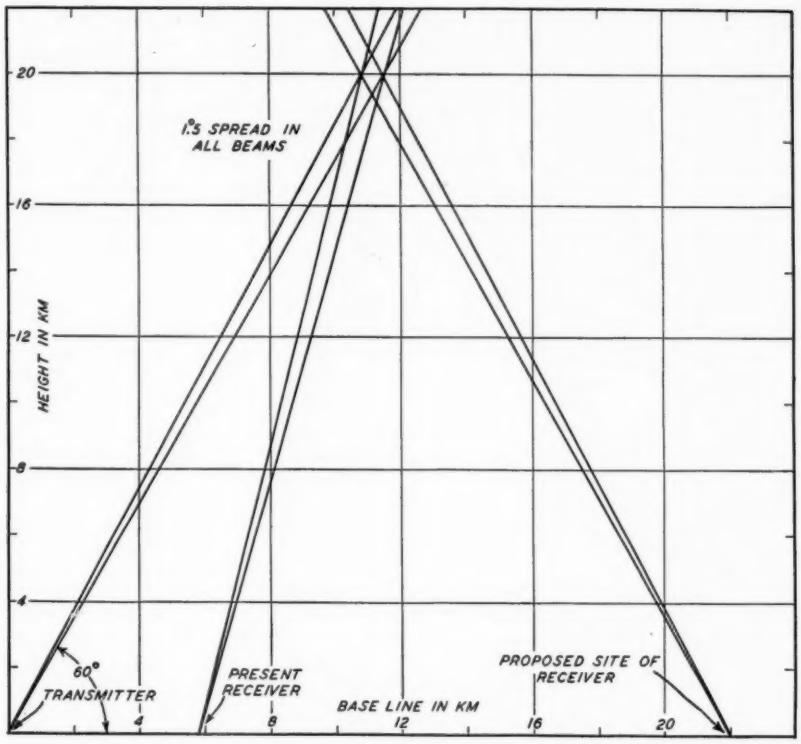

FIG. 1. Geometry of search-light experiment.

unit-volume of air is given by

$$K = 32\pi^3(\mu-1)^2/3N\lambda^4, \qquad (1)$$

TABLE I. *Response calculations from a 24-inch high intensity search-light.*

WAVE-LENGTH μ	ENERGY IN BEAM FOR 0.04-μ BAND WATTS	$E_{s\lambda}\times10^{25}/N$ WATTS	RESPONSE OF (Cs−O)-CELL TO $E_{s\lambda}(\mu a$, PEAK A.C.) $\times10^{22}/N$
0.30	7.56	3.51	0.05
0.34	12.92	3.74	3.06
0.38	48.0	8.87	5.79
0.42	41.8	5.17	2.06
0.46	31.4	2.71	0.606
0.50	31 4	1.94	0 372
0 54	31 4	1.43	0 300
0 58	28 3	0.95	0.276
0.62	25.8	0.68	0.280
0.66	24.6	0.49	0.250
0.70	24.6	0.41	0.250
0.74	24.6	0.31	0.201
0.78	11.7	0.12	0.075
0.80	29.2	0.27	0.160
0.85	30.7	0.23	0.255
0.90	29.2	0.17	0.180
0.95	27.7	0.13	0.085
1.00	26.9	0.10	0.025
1.05	26.9	0.08	0.004

where N = number of molecules per cc, λ = wave-length of radiation in cm, and μ = index of refraction of the scattering gas. For air $(\mu-1)=\alpha N$ where α for air is 1.08×10^{-23}. The value of α varies slightly with wave-length but in these calculations is considered constant. Eq. (1) can then be written

$$K = 330\alpha^2N/\lambda^4. \qquad (2)$$

If the radiation incident upon the scattering volume is $E_{o\lambda}$, the radiation scattered is $E_{s\lambda}$ and is then given by

$$E_{s\lambda} = E_{o\lambda}330\alpha^2N/\lambda^4. \qquad (3)$$

When $E_{o\lambda}$ is taken as the energy in the search-light beam at any particular height passing through a cross section of the beam, $E_{s\lambda}$ is the total radiation scattered in all directions by each cm of traverse of the beam. The third column of Table I gives $E_{s\lambda}$ for the assumed source. The fourth column of Table I gives the response of a calibrated (Cs−O)-photoelectric cell to the

energy listed in the third column. The integrated response for all wave-lengths for this cell in microamperes is then given by

$$I_s' = 1.43 \times 10^{-21} E_{s\lambda} N. \quad (4)$$

The energy incident upon the cell is obtained from the geometry of Fig. 1 upon the assumption that a negligible amount of energy is absorbed by the atmosphere. Taking into account the distribution of the scattered light according to the cosine-law, Eq. (4) becomes

$$I_s = 1.13 \times 10^{-20} l\omega(1+\cos^2\beta) E_{s\lambda} N, \quad (5)$$

where l = length of beam viewed by the receiver, ω = solid angle subtended by the mirror of receiver, and β = angle between beam and direction from which it is viewed.

It is assumed that the volume intercepted by the beam and receiver is a cylinder of diameter equal to that of the beam. The transmitter was set at 60° with the horizontal with a base-line of 5.8 km. Assuming a field of 1.5° for a 30-inch receiving mirror and 100 percent optical efficiency, column 4 of Table II gives the expected response of the system with height. The values for temperature and N are taken from Hulburt's paper[3] and were derived from Humphrey's tabulations.

Experimental Results

A 24-inch high intensity carbon-arc search-light with an aluminized mirror was used as a transmitter. The carbons were 11 mm outside diameter, and the arc was maintained at 75 amperes and 50 volts. The measured angular spread of the beam was approximately one degree. The modulation of the beam was accomplished by a Venetian-blind type of shutter driven through a gear-reduction by a 60-cycle synchronous motor. The shutters were mounted above the beam on a frame, which also held the search-light. The search-light could be tilted and rotated to any desired angle.

Table II. *Response of* (Cs-O)*-cell at focus of 30-inch mirror to radiation scattered from a search-light beam at various heights.*

Height km	Temperature °C	N, molecules per cc $\times 10^{-17}$	I_s, microamperes peak a.c. $\times 10^{7}$
2	− 4.16	210	4.0
4	−15.3	170	3.4
6	−29.3	137	3.5
8	−43.6	110	3.0
10	−54.2	86.0	2.8
14	−55	47.5	1.8
18	−55	25.2	1.1
22	−55	13.7	0.53
26	−55	7.32	0.28
30	−55	3.95	0.16

The transmitter was located on the outskirts of Washington, D. C., on the grounds of the Department of Terrestrial Magnetism, and the receiver was located 5.8 km distant at Kensington, Maryland. The receiver was composed of a 30-inch diameter, 12-inch focal length, aluminized search-light mirror, at the focus of which the photoelectric cell was located. The mirror was mounted on a frame to allow any section of the beam to be scanned, with an arrangement for determining the angle. Two photoelectric cells were used, one a (Cs−O)-double-ended cell designed to have a low leakage and dark current and the other a (Cs−Sb)-cell of special construction to increase the optical efficiency of the system and also to have very low leakage. In the case of the (Cs−O)-cell the cathode surface acted as a focal-plane diaphragm effectively to limit the field of the receiver to 1.5°, while in the case of the (Cs−Sb)-cell a re-entrant window served a similar purpose.

The photo-cell and the first stage of amplification using an electrometer tube were mounted together above the receiving mirror and the output-signal was amplified by an amplifier tuned to ten c.p.s. The circuit diagram and the method of calibration are described in Fig. 1 of the preceding paper in this Journal. The low frequency was desirable both because of mechanical considerations in modulating the source and because the lower frequencies lower the limit of measurement under certain conditions.

The (Cs−O)-cell was calibrated with the aid of a standard ribbon-filament lamp loaned to us by the National Bureau of Standards, and the (Cs−Sb)-cell was compared with the (Cs−O)-cell for an indirect calibration.

The receiver-sensitivity was limited in the case of the (Cs−O)-cell by its own dark current and in the case of the (Cs−Sb)-cell by the background light from the night sky. This background light produced a photoelectric current of approximately 2×10^{-12} ampere in both cases. This

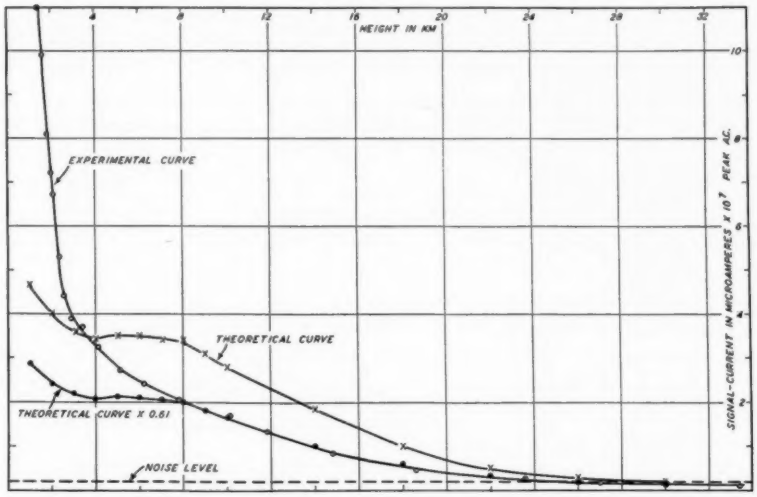

FIG. 2. Theoretical and experimental response of receiver to scattered light from search-light beam.

limiting sensitivity due to the shot-effect in the current in the photoelectric cell caused by light from the night sky cannot, therefore, be appreciably changed unless the background current can be reduced. Since the spectrum of the light from the night sky is very broad this may be accomplished principally by using a cell-filter combination sensitive to the part of the spectrum between 4000A and 3000A, thus decreasing appreciably the energy due to the light from the night sky effective in producing a photoelectric current. We were unable to employ a filter without excessive loss of sensitivity because of the large f number of our mirror and could not take advantage of this effect. The general problem of the limit of light measurements is discussed in the preceding paper of this Journal.

Observations were made only on cloudless (visually) moonless nights. The search-light beam was always directed at 60° with the horizontal and the beam was scanned by varying the angle of the receiving mirror. The results of observations made on September 8, 1939, are shown in Fig. 2, together with the calculated values given in Table II. The calculated values have been multiplied by 0.61 to bring the upper part of the curve in agreement with the experimental results. The apparent efficiency of 61 percent is adequately accounted for by the fact that the optical efficiency of the receiver was known to be less than 70 percent and by the fact that some atmospheric absorption probably occurred below four km. Since these two factors could not be accurately measured it is considered that the absolute values of the received signal are in very good agreement with the calculated ones.

From the excellent agreement above eight km it is seen that, as would be expected, there was no appreciable deviation from the assumed atmosphere. The sharp rise at two km is assumed to be due to low-lying haze which caused particle scattering. The upper part of this haze varied considerably from night to night over a range from four to seven km, but a sharp boundary seemed always to be maintained at two km for the principal part of this haze bank. This phenomenon was also observed by Hulburt.[3]

The noise level in Fig. 2 is shown by the dotted line. The calculated and experimental limits agreed very well. It was found that a signal could just be detected at 40 km.

Many apparently clear nights actually proved to be considerably disturbed by the presence of invisible clouds and absorbing material. Many of these clouds were transparent, that is, the beam was perfectly defined on both sides of the cloud. Such clouds occurred principally in the

10- to 12-km region. Fig. 3 shows a curve taken on August 22, 1939, up to ten km, together with a good example of a transparent cloud at 12 km. The dotted part of the curve was estimated from deflections of the cathode-ray oscillograph since the cloud was moving too rapidly to allow a complete delineation of the curve.

Other nights showed the presence of a varying absorption or scattering of fairly rapid period. Fig. 4 shows the signal-variation at 16 km on the night of September 5, 1939, together with the variation in noise level.

In the present work no attempt was made at great accuracy. The accuracy of the observations could be made very good by using a monitor receiver set a height of four km, since it is felt that a varying absorption takes place below this height and because of known variations in transmitter-output. It would also be very advantageous to use a receiver with a lower f number in order to improve the optical imagery and to allow the use of filters. If measurements are to be made at great heights it would be necessary to use a longer base-line in order to increase the resolution. The effect of changing the base-line is shown in Fig. 1. However, changing the base-line from 5.8 to 22 km also decreases the sensitivity about tenfold.

It is possible to predict the increase in height which would be possible with an improved system. If a 60-inch aluminized receiver and a 60-inch aluminized transmitter with 18-mm carbons were used, together with a (K−O)-cell at the receiver and a 22-km base-line, it would be possible to gain at a maximum a factor of 100 in sensitivity. This should make observations possible to a height of 70 to 80 km. Greater heights with poor resolution would be possible from the 5.8-km base-line.

The ultimate usefulness of this method is unknown, but it appears that it may be adapted to the study of a number of important atmospheric problems. Such problems are the occurrence with height of water-vapor, turbulence, winds, dust, fluorescence, and absorption. One of the most interesting problems of meteorological importance is the study of the distribution of ozone in the atmosphere. Since ozone has a strong absorption-band from about 2500A to 3300A, it would be possible to compare the scattering from this region with that above 3400A. This can be done by the use of two receivers using a sodium and potassium oxide cell, respectively.

Acknowledgments

The authors are indebted to Dr. M. A. Tuve and Dr. O. R. Wulf as well as to many other colleagues in Washington for discussion and loan

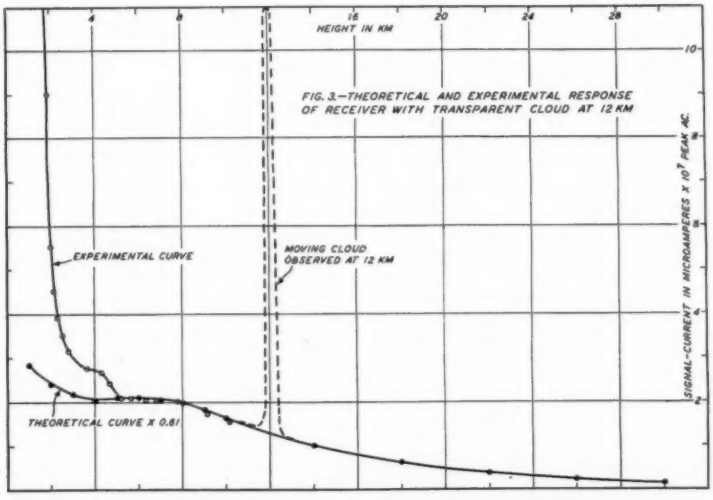
HEIGHT IN KM
FIG. 3.—THEORETICAL AND EXPERIMENTAL RESPONSE OF RECEIVER WITH TRANSPARENT CLOUD AT 12 KM
EXPERIMENTAL CURVE
MOVING CLOUD OBSERVED AT 12 KM
THEORETICAL CURVE X 0.81
SIGNAL-CURRENT IN MICROAMPERES X 10⁷ PEAK AC

FIG. 3.

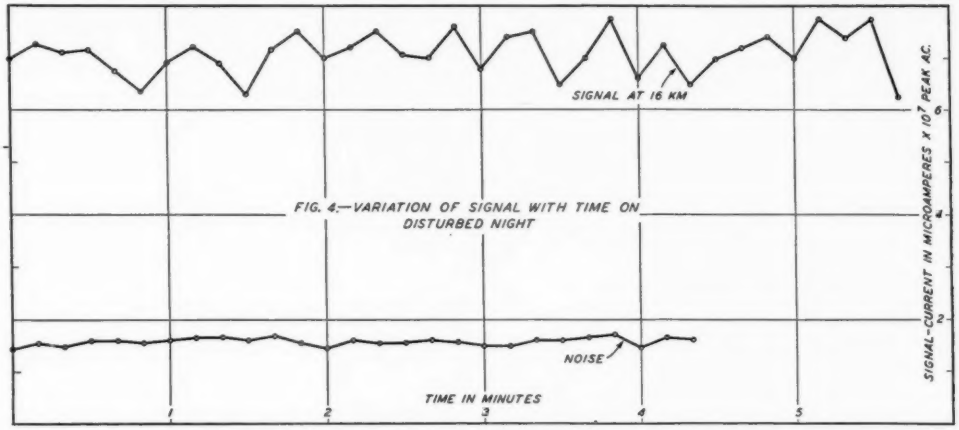

FIG. 4.

of apparatus during the research. They are also indebted to the Bell Telephone Laboratories for the photoelectric cells used. Finally, they are especially indebted to P. R. Bassett of the Sperry Gyroscope Company for the search-light mirrors used in both the transmitter and receiver.

Number of Quanta Required to Form the Photographic Latent Image as Determined from Mathematical Analysis of the H and D Curve

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(Received October 19, 1939)

THIS note is supplementary to the paper of the same title by J. H. Webb.[1] Put very briefly, the point at issue is that if we have an emulsion of identically shaped grains of identical sensitivity, the statistical variation in quantum absorption leads to characteristic curves, as shown in Fig. 3 of Webb's paper, wherein the lower the number of quanta necessary to make a grain developable the lower the slope of the theoretical characteristic curve. The curve with the minimum possible slope is obtained when the number of quanta necessary is one. But, as indicated here in Fig. 1, some early results by Toy,[2] on grains which were carefully selected for geometrical equality, give characteristic curves with slopes substantially less than the theoretical minimum. It is possible to make an estimate of the sensitivity of these grains if the Poisson distribution is replaced by a Gaussian distribution approximating to it.[3] Details of the calculation are hardly worth describing since fractions of a quantum are physically meaningless, but it may be worth mentioning that the results of such a calculation indicated that $\frac{1}{10}$ quantum was sufficient to make Toy's class *d* grains developable, and $\frac{1}{5}$ quantum the class *c* grains. It is extremely likely therefore that nearly all the spread of Toy's curves along the log exposure axis is due to variation in sensitivity from grain to grain. For the grains in question, the spread seems to be too great, also, for it to be due merely to a variation in thickness (and therefore in absorption) of grains of the same surface area. We may conclude, therefore, that at least some cases must exist where Webb's suggestion that geometrically identical grains of a given emulsion are not all of equal sensitivity is true. This was also maintained by Toy in the paper quoted, and in other papers, on somewhat different grounds.

[1] J. H. Webb, J. Opt. Soc. Am. **29**, 309 (1939).
[2] F. C. Toy, Phil. Mag. **44**, 364 (1922).
[3] T. C. Fry, *Probability and Its Engineering Uses* (D. Van Nostrand Co. Inc., New York, 1928), p. 237.

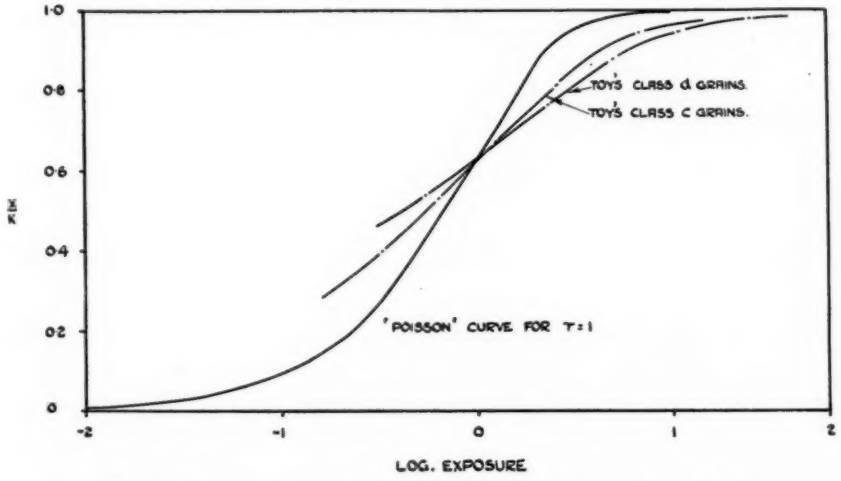

FIG. 1. Illustrating H and D curve of theoretical minimum slope (full line) and experimental curves of lower slopes. The experimental curves have been adjusted to intersect the theoretical curve at log exposure equal to 0.

The Visibility of a Thin Wire

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(Received October 26, 1939)

A THIN dark wire set against a background not too bright, say 0.015 candle/cm², can be detected by the unaided eye when it subtends an angle of three or four seconds. This seems a rather small limit, since two bright line-sources are not resolvable unless they subtend about sixty seconds or more. Although the two cases are not strictly comparable, it seems worth while to calculate the brightness-ratio between the center of the vanishing image of the wire and the background, to see what brightness-difference the eye can detect in the case of a narrow and shallow minimum on an otherwise uniform field.

The calculation is not hard. Suppose the wire is situated on the optic axis of a telescope, subtending an angle β. Let λ be the mean effective wave-length, and a the width of a rectangular aperture at the telescope objective, adjusted till the image of the wire is just perceptible in the telescope field. Imagine the background on each side of the wire to be divided into strips of infinitesimal width. According to well-known diffraction theory, a strip of width $d\varphi$, situated at angular position φ—measured from the optic axis—produces in the focal plane a diffraction pattern such that the intensity at angle θ from the optic axis is

$$dI = Cd\varphi \sin^2 x/x^2,$$

where $$x = \pi a(\varphi - \theta)/\lambda.$$

The resultant intensity at θ is the integral, taken from one edge of the bright field to $-\beta/2$ and then from $+\beta/2$ to the other edge of the field.

$$I_\theta = C\int_{\varphi_1}^{-\beta/2} \frac{\sin^2 x}{x^2} d\varphi + C\int_{+\beta/2}^{\varphi_2} \frac{\sin^2 x}{x^2} d\varphi$$

$$= C\int_{\varphi_1}^{\varphi_2} \frac{\sin^2 x}{x^2} d\varphi - C\int_{-\beta/2}^{+\beta/2} \frac{\sin^2 x}{x^2} d\varphi.$$

In the first integral we replace the limits by $-\infty$ and $+\infty$. This is justifiable because the brightness of a moderately wide field is not changed by making it still wider. We also change the variable of integration from φ to x. Since

$$\int_{-\infty}^{+\infty} \frac{\sin^2 x}{x^2} dx = \pi,$$

a known form, the expression simplifies to

$$I_\theta = \frac{C\lambda}{\pi a}\left[\pi - \int_{x_1}^{x_2} \frac{\sin^2 x}{x^2} dx\right].$$

The second integral cannot be readily evaluated, but it becomes manageable if we put $\theta = 0$, giving I for the center of the field, i.e., the center of the vanishing image of the wire. Then

$$I_0 = \frac{C\lambda}{a\pi}\left[\pi - \int_{-\pi a\beta/2\lambda}^{+\pi a\beta/2\lambda} \frac{\sin^2 x}{x^2} dx\right].$$

The integral is equal to the area of a central strip, of width $\pi a\beta/\lambda$ under the curve $\sin^2 x/x^2$. The width of this curve between the two inner minima is 2π. Experiments using the telescope show that the wire vanishes when $\beta = 0.05\lambda/a$ to $0.06\lambda/a$. Therefore the width $\pi a\beta/\lambda$ is only a small fraction of the width of the central segment of the diffraction curve, and the area in question is closely equal to the width $\pi a\beta/\lambda$ times the ordinate at the center, which is unity. Therefore

$$I_0 = \frac{C\lambda}{\pi a}\left[\pi - \frac{\pi a\beta}{\lambda}\right] = \frac{C\lambda}{a}\left[1 - \frac{a\beta}{\lambda}\right]$$

$$= \frac{C\lambda}{\pi a}[1 - 0.06].$$

This gives $\Delta B/B$ = about 6 percent (B being the intensity of the background and $B - \Delta B$ as that at the center of the image), as an expression for the sensitivity of the eye to contrast under these conditions.

Probably the conditions for greatest contrast sensitivity are those presented by a modern visual photometer. There it is generally agreed that $\Delta B/B$ is about 1 percent to 2 percent.

A New Radiation Pyrometer

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(Received October 25, 1939)

A pyrometer is described which measures with a thermopile and galvanometer the emission of energy in an isolated narrow band of wave-lengths centered at 8.8μ. These radiations are isolated by successive reflections from quartz crystals. The residual-ray apparatus used to isolate the 8.8μ band is arranged in a manner to make it compact, to avoid aberrations and, owing to the polarization of the near infra-red radiations, it is more effective than earlier models of residual-ray apparatus in eliminating the undesired short wave-length infra-red energy. Because the band lies in the infra-red the pyrometer can be used to determine low as well as high temperatures. The instrument is particularly suited for work in the temperatures range 0°C±100°C. Ordinary temperatures are easily determined to 0.1°C. The band of radiations used by the instrument falls in a region of the infra-red spectrum where the atmosphere is very transparent. Accordingly, in most applications (meteorological and astronomical applications excepted) there is practically no absorption in the optical path. The procedure of making temperature measurements is described. A new temperature scale is employed. Tables are supplied for reducing the observations to the ordinary centigrade scale. The instrument measures surface temperatures (when the emissivity of the surface is known) without disturbing radiation transfer or convective heating and cooling at the surface. Many applications of the instrument are concerned with surfaces which radiate as blackbodies.

INTRODUCTION

HERETOFORE, there has been available no temperature measuring instrument which operates at a distance and is suitable for the determination of ordinary temperatures.[1] This paper treats of a new radiation pyrometer which is intended primarily for temperatures ranging from −100°C to +100°C. The word pyrometer may be somewhat of a misnomer considering that the instrument has been used to determine the temperature of liquid oxygen but the word is justified by the use of the instrument to measure the temperature of the sun.

A body being heated becomes "infra-red hot" before it becomes "red hot." Accordingly, the new pyrometer uses as the thermometric property a band of radiation in the infra-red (at 8.8μ) as contrasted with the ordinary optical pyrometer which uses a band of red radiation (at 0.6μ). This infra-red band falls within the region of high transparency in the atmosphere, 8–14μ, so that the pyrometer may be used at a distance remote from the object being tested without necessitating corrections for absorption in the optical path.

The band of radiation used in this pyrometer is the 8.8μ residual-ray band of quartz.

RESIDUAL RAYS

As the wave-length increases the reflection of quartz decreases from the value it exhibits in the visible until it becomes vanishingly small around 7.3μ. Beyond this wave-length lies a band of high reflection, the so-called band of metallic reflection of quartz.

E. F. Nichols, in 1896, discovered the phenomenon of the metallic reflection of a quartz crystal surface.[2] After this discovery, Rubens and Nichols published a paper describing a general method to filter out the major part of the heat spectrum emitted by a hot body and separate bands of long wave-length infra-red radiations from it.[3] The separation is effected by four or five successive reflections from crystal surfaces. The experimental arrangement they used is shown diagrammatically in Fig. 1 where a is a hot body, b is a concave mirror imaging a at S after four reflections from crystal surfaces p_1 to p_4.

We may take five successive reflections from quartz crystal surfaces as an example to illustrate the effectiveness of the filtering thus obtained.

[1] J. D. Hardy and G. F. Soderstrom, Rev. Sci. Inst. **8**, 419 (1937). These authors describe an improved apparatus for measuring the total radiation from surface at room temperatures. The instrument measures the total radiation emitted by the surface: no correction for the absorption in the optical path is necessary as the instrument is always used near the tested surface.

[2] E. F. Nichols, Ann. d. Physik **60**, 401 (1897).

[3] H. Rubens and E. F. Nichols, Ann. d. Physik **60**, 418 (1897).

After five reflections at quartz surfaces the energy distribution in the spectrum of the source is drastically changed. For normal incidence we may take the reflectivity of quartz throughout the spectral region from 0.5 to 8μ as being approximately 1/25. Accordingly, the energy remaining in the spectral region 0.5 to 8.0μ after five reflections will be only $(1/25)^5$ or about 1/10,000,000 of incident energy. The energy at wave-lengths greater than 9.6μ is similarly attenuated. The energy remaining in the metallic band, after five reflections will be $(0.9)^5$ or about 0.6 of the incident energy. This "transmission factor" of 0.6 is approximately the same as is the transmission factor in the visible for a fast camera lens. Because of the manner in which the filtering method operates the resulting radiations obtained are known as *residual rays*.

The residual-ray method yields wave-lengths from 6.7μ to 150μ. The wave-lengths obtained using various crystals and experimental arrangements are given in the literature.[4] The wave-lengths of the residual rays for a given crystal vary somewhat, depending on the number of reflections, the orientation of the surfaces relative to the crystallographic axes, whether the incident heat spectrum is polarized or not, what screens and windows are used in the optical path and also, but to a minor degree, on the angles of incidence at which the crystal surfaces are used. Fig. 2 gives the spectral distribution in the 8.8μ residual rays of quartz. The apparatus with which the curves were obtained and the experimental conditions are described on page 525. Using a similar set-up one should closely reproduce this distribution curve. (It is intended in the future to remeasure under uniform conditions the distribution curves of all the important residual rays.)

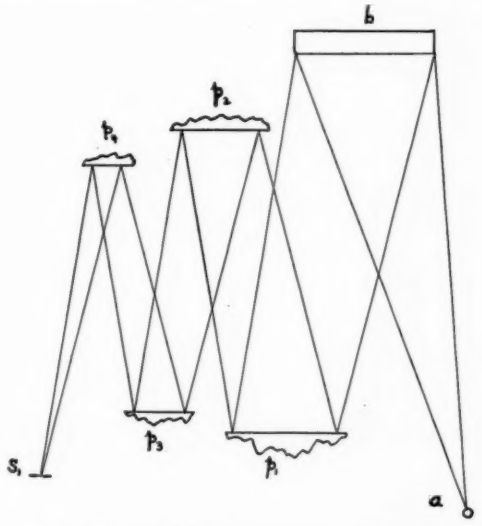

FIG. 1. Rubens-Nichols arrangement for obtaining residual-rays.

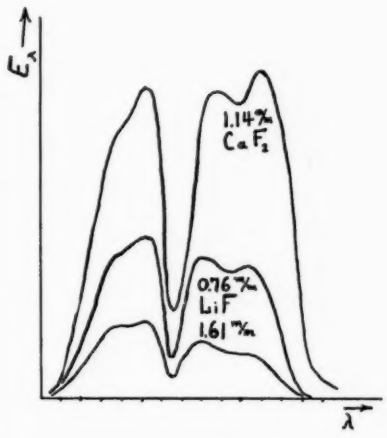

FIG. 2. Energy distribution in the 5-crystal quartz residual-ray band with various filters. Note: the scale of abscissa is the same as that of Fig. 5. See also Fig. 7.

The comparative ease with which the residual-ray method yields workable beams of radiation especially in the far infra-red, accounts for its frequent use in physical investigations. Especially notable was the use of the residual rays of quartz, fluorite and rocksalt to study the isochromatic emission curves of the blackbody. It was primarily on the results of this work that the quantum theory was experimentally established.[5]

RUBENS-NICHOLS RESIDUAL-RAY APPARATUS

The manner of arranging the residual-ray apparatus due to Rubens and Nichols is shown

[4] T. Liebisch and H. Rubens, Preuss. Akad. Wiss. Berlin, Ber. **16**, 198, 876 (1919); M. Czerny and H. Röder, "Fortschritte auf dem Gebiete der Ultrarottechnik" and also Frank Matossi, "Ergebnisse der Ultrarotforschung" Ergebnisse der exakten Naturwissenschaften Vol. 17, 1938.

[5] H. Rubens and F. Kurlbaum, Ann. d. Physik **4**, 649 (1901).

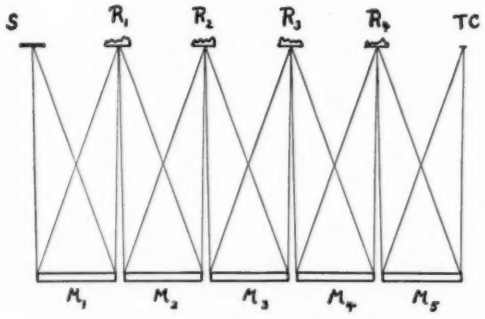

FIG. 3. Small-crystal residual-ray apparatus.

in Fig. 1. Let us consider some of the characteristics which make it effective for use in the far-infra-red region of the spectrum. After the near-infra-red radiation is disposed of, there is, at best, but little energy remaining in the far-infra-red residual-ray bands owing to the inverse fourth power dependence of the energy on wave-length. However, the far-infra-red energy available using the residual-ray method is comparatively more intense than that available by means of the prism or grating method since one can use the energy emitted from an extended area of the source with the residual-ray apparatus as contrasted with the narrow strip of the source used by a prism or grating monochromator. In addition, one can more easily collect and use the radiations emitted into a large solid angle with the residual-ray method.

With the Rubens-Nichols apparatus it is not easy to pass from one residual-ray band to another. The whole optical system is necessarily disturbed when four of the five mirrors which constitute it are changed. Also, after the crystal mirrors are changed it takes the instrument a long time to re-establish the necessary degree of thermal equilibrium.

SMALL-CRYSTAL RESIDUAL-RAY APPARATUS

The use of the residual-ray method is facilitated by the modified arrangement shown in Fig. 3. This apparatus has been described elsewhere[6] but it is appropriate to describe some of its features here. M_1 to M_5 are five spherical mirrors coated with aluminum. R_1 to R_4 are the crystal reflectors. S is the source of radiation and TC the thermocouple. In contrast with the apparatus shown in Fig. 1 this modified instrument requires only small-crystal surfaces ($\frac{3}{4}''$ as compared with $4''$ or so in diameter). With the modified arrangement the crystals for one wave-length can be replaced by crystals for another wave-length in a short time without sensibly disturbing the thermal equilibrium established in the main part of the apparatus. Because the residual-ray mirrors are small, they quickly assume thermal equilibrium with the main part of the apparatus. The apparatus is enclosed in a box with six small apertures, four for the crystals and two for the source and thermocouple. These small apertures are covered with thin lacquer films so that the main part of the optical path is undisturbed by the changing of residual-ray mirrors and so it may be kept dry.

This modified apparatus has proved useful for making preliminary studies of optical properties of various materials in the far infra-red region.[7] Preliminary investigations of the molecular spectra of gases have been made in the spectral region 20–150μ using this apparatus.[8] In this connection it is of interest to note that the frequency resolution obtained in these spectra is comparable with that obtained in similar preliminary investigations in the near infra-red using prism spectrometers. Such spectra, obtained with residual rays, are useful in determining if a band observed in the near infra-red with a prism or grating monochromator is a fundamental band or whether it is a harmonic of a band lying beyond 20μ.

The small-crystal apparatus shown in Fig. 3 is no improvement over the older arrangement in respect to aberrations introduced in the image at the receiver. These aberrations would result in a serious loss of illumination at TC if one did not use an oversized source of radiation at S. The large radiation source ($\frac{3}{4}''$ diameter) necessary to compensate for aberration puts no serious restriction on the use of this instrument in the laboratory since a large source of radiation is available in the Welsbach mantle. However,

[6] John Strong, Phys. Rev. **37**, 1565 (1931); *Procedures in Experimental Physics* (Prentice-Hall, New York, 1938), p. 383.

[7] John Strong, Phys. Rev. **38**, 1818 (1931).

[8] John Strong and S. C. Woo, Phys. Rev. **42**, 267 (1932).

for measuring the radiation emitted by a body of restricted angular size such as the sun or moon, the aberrations become serious. Accordingly, the apparatus shown in Fig. 3 has been further modified so that aberrations are reduced, even smaller crystals ($\frac{1}{2}''$ diameter) are adequate and the instrument is more compact and effective.

The New Residual-Ray Apparatus

The new residual-ray apparatus is shown in Fig. 4.[9] A heat spectrum from any object is focused on the aperture A. The energy in a band in this spectrum (determined by the residual-ray crystals) is received and measured with a thermocouple receiver at TC. Immediately behind aperture A and on top of the turret R_1 is a crystal-holder plate with an elliptical hole in it ($\frac{3}{8}''\times\frac{1}{2}''$). This plate is so positioned that the first residual-ray mirror placed on it reflects the diverging beam from A to the concave aluminized spherical mirror M_1. This mirror reflects the beam to the second residual-ray surface placed on turret R_2 and thence laterally through apertures in the sides of R_2 and R_3 to a third crystal surface on top of turret R_3. The beam is brought to focus by M_1 midway between R_2 and R_3. The produced image is astigmatic. The astigmatism arises on account of using the spherical mirror "off-axis."

The diverging beam from this astigmatic image after reflection by a crystal on R_3 falls on a second spherical mirror, M_2 which again reflects and converges it. After reflection from a fourth residual-ray crystal on R_4 the beam comes to a focus between the turrets R_4 and R_5. This is a sharp image because the arrangement of M_1 and M_2, taken together, is such that aberrations introduced by the spherical mirror M_1 are corrected in first approximation when the beam is reflected by M_2.

After reflection from a crystal on R_5 the residual rays are finally focused by an elliptical mirror M_3 on the active junction of a compensated vacuum thermocouple. The use of an elliptical mirror is suited for forming this focus since the reflected conjugate image positioned between turrets R_4 and R_5 is small and all points in it lie close to the optical axis of the elliptical mirror.

The intensity of the residual-ray wave-lengths is determined by means of the thermopile and connected galvanometer. The thermopile receiver is "blackened" with finely powdered crystalline quartz and the vacuum is maintained by calcium filings contained in a small Pyrex bulb connected to the communicating brass tube at V. The thermopile housing is pivoted at P so that it is possible to adjust the position of the active receiver along the axis of the elliptical mirror by simply rotating the evacuating tube and clamping it by means of a clamp on the top plate. The top and bottom plates are $4\frac{1}{4}\times5\frac{1}{4}$ inches. The diameter of M_3 (as well as M_1 and M_2) is 47 mm. The mirror cells for M_1, M_2, and M_3 are attached to the bottom plate of the apparatus and each cell is fitted with four adjusting screws. Visible light may be used for adjusting the optical system of the residual-ray apparatus. One first cuts, to the proper length, the four separators connecting the plate to which the turrets are attached and the plate to which the mirrors are attached. The turrets are rotated, clamped and marked at the correct azimuth on

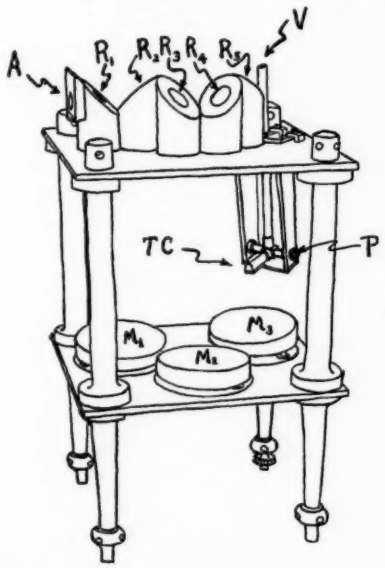

Fig. 4.

[9] This apparatus was described at the meetings of the American Physical Society in San Diego, 1938, and Washington, 1939. See Phys. Rev. **54**, 242 (1938); **55**, 1114 (1939).

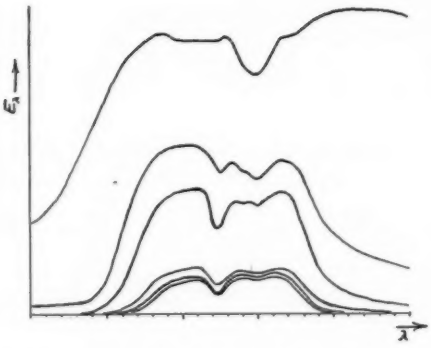

FIG. 5. Residual-ray bands of the hot glass envelope of a tungsten lamp, obtained with 0 (top curve), 1, 2, 3, 4 and 5 (lowest curve) quartz reflections.

the top plate. The construction angle of the crystal-holder plates R_2 to R_5 is 48° 40′. The angle of R_1 is 47° 12′. Fine adjustment of these angles is achieved by filing. Making the adjustments is facilitated by small aluminized glass plates fitted to the crystal apertures. It is necessary to make the adjustments of the instrument only once. Thereafter, the apparatus may be dismantled and reassembled without disturbing the adjustment. After adjusting the apparatus three brass cylinders fit in grooves between the top and bottom plates of the apparatus and so enclose the optical path. These cylinders are slightly larger in diameter than the mirror cells M_1 to M_3. The grooves in the lower plate, where the cylinders fit, are shown in Fig. 4 encircling each of the mirror cells. A chamber fitted to the tube enclosing the thermopile and elliptical-mirror assembly contains magnesium perchlorate to dry the air and preserve the KI window of the thermopile. A filter holder, not shown, fits between turrets R_4 and R_5.

The apparatus as shown in Fig. 4 (but with shorter legs) is mounted in an aluminum box of dimensions 5×6×8 inches. The space between the apparatus and this box is thermally insulated with aluminum foil. This aluminum box is attached to a second box containing an f: 3 off-axis mirror telescope. Aperture A is at the focus of this telescope. Radiations focused by the telescope pass through a small hole in the aluminum box to reach A. Both the aluminum box and the attached telescope are, in turn, mounted in a wooden box. The space between the apparatus and the wooden box is thermally insulated with more aluminum foil. Finally, the wooden box is covered with bright sheet aluminum. This arrangement of a box in a box effectively insulates the apparatus thermally. This is necessary if it is to be used out-of-doors at night and in sunshine and wind. It is as if the thermopile chamber were surrounded by a substance having an extremely low heat diffusivity. This procedure has proved to be very effective: in operation the deflection is very steady and the drift is small. When working out-of-doors with the instrument exposed to full sunlight during the day and without further protection at night the drift is so small that it is unnecessary to move the galvanometer scale.

The arrangement shown in Fig. 4 is different from the earlier one owing to the use of the residual-ray crystal surfaces at a larger angle of incidence and in the folding of the plane of incidence of the optical system so that there is an angle of 90° between the plane of incidence at the first concave mirror M_1 and the plane of incidence at the second and third mirrors. The results of this folding are to make the instrument compact and to reduce aberrations. Also, the folding of the plane of incidence together with the use of the residual-ray reflecting surfaces near their polarizing angle results in an attenuation of undesired near infra-red radiation by a factor of ten over that achieved when the residual-ray crystals are used at normal incidence.

POLARIZATION EFFECTS

The angle of incidence on the crystal mirror R_1 varies for different rays striking it owing to the large angular aperture of the apparatus. However, we may regard the angle of incidence as being, on the average, 48° 40′. For this angle the reflectivities at a quartz surface for wavelengths from 0.5 to 7μ are approximated by 0.5 percent and 10.9 percent (for the two azimuths of the electric vector) as contrasted with 4.15 percent for normal incidence. The action of turret R_1 is partially to polarize the near infra-red radiation. The band of radiations reflected metallically by quartz is, of course, not polarized. The action of turrets R_2 and R_3 is, also, to yield polarized radiation. The azimuth

of the polarization produced by turret R_1 is at an angle of approximately 45° with the polarizing azimuth of turrets R_2 and R_3. There is, therefore, no great attenuation as a result of the polarization of turret R_1.

However, the azimuth of the polarization produced by the turrets R_4 and R_5 is, crossed in respect to the azimuth of the polarization produced by turrets R_2 and R_3 owing to the folding of the plane of incidence, mentioned above. In the near infra-red the effect of crossed azimuths is the same as the effect of introducing crossed Nicols in a beam of light. The near infra-red energy is greatly attenuated. The band of metallic reflection is not correspondingly attenuated since it is not polarized. This polarization effect has been employed by Czerny with two residual-ray crystals.[10] But here we have five rather than two reflections.

To evaluate this polarizing effect obtained in the new apparatus we compare the near infra-red attenuation factor of the turrets R_2 to R_5,

$$(10.9\times10^{-2})^2(0.5\times10^{-2})^2\sim3\times10^{-7},$$

with the attenuation factor for normal incidence obtained in the older apparatus,

$$(4.15\times10^{-2})^1\sim3\times10^{-6}.$$

We see that the polarization effect yields a tenfold additional attenuation. Combining this factor with the attenuation produced by the residual-ray mirror on turret R_1 the total attenuation of the near infra-red radiation by the five crystals comes out to be approximately

$$2\times10^{-8}.$$

Study of the 8.8μ Residual-Rays of Quartz

In contrast with this attenuation factor for the 0.5 to 8μ radiations, let us see how the radiations in the metallic band, 8.2 to 9.4μ, are preserved. Various spectra of the emission of the hot envelope of an incandescent tungsten lamp were determined. See Fig. 5. The lowest curve in Fig. 5 was determined first with five quartz crystals. The other curves with, respectively, 4, 3, 2, 1 and 0 quartz reflections were then determined in the order given. The turrets on which quartz crystals were placed are listed in Table I. Aluminized glass plates were placed on the other turrets. Fig. 6 shows the arrangement used to obtain these curves. L is the hot lamp envelope. S_1 is the entrance slit of a grating spectrometer. The collimator of the rays consists of the mirrors M_1 and M_2. G is an eschelette grating. This grating was kindly loaned by Professor R. W. Wood. M_3 and M_4 are the optical parts of the telescope of the new pyrometer. S_2 is a slit attached to the entrance

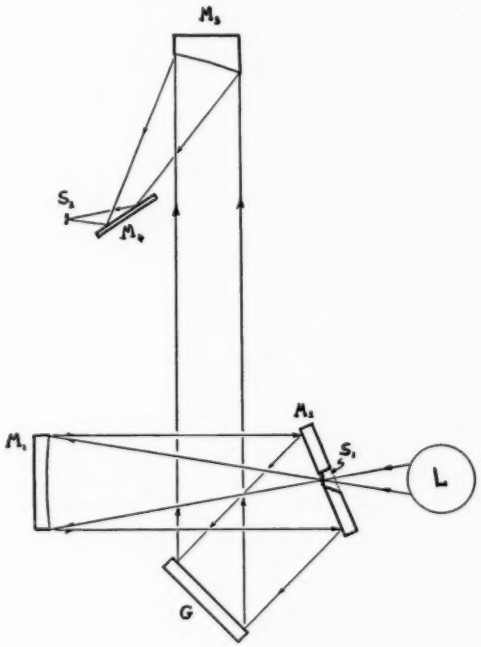

Fig. 6. Experimental arrangement used to get curves shown in Figs. 2, 5 and 7.

aperture of the residual-ray apparatus (A in Fig. 4). The telescope of the pyrometer is pointed at the eschelette grating and the distribution in the residual-ray band is obtained by rotating the grating, G.

The overlapping second-order short wavelength energy, attendant with the use of a grating, was not very intense owing to the low temperature (100°C) of the source. However, it was possible partially to account for this

[10] M. Czerny, Zeits. f. Physik **16**, 321 (1923).

TABLE I.

Turrets					$d_{100}-d_0$ mm	Ratio
R_1	R_2	R_3	R_4	R_5		
Q	Q	Q	Q	Q	420	0.87
Al	Q	Q	Q	Q	484	0.84
Al	Q	Q	Q	Al	575	0.78
Al	Al	Q	Q	Al	734	

energy by using a β-MgO crystal as the shutter. This shutter is transparent for $\lambda < 8\mu$.[11]

Spurious results come in with the simple arrangement used owing to the fact that the telescope and slit, being a part of the residual-ray apparatus, were not exposed to the heating by the electric lamp source whereas the temperature of the collimator and grating were being slowly elevated. Also, during the measurements, the circulation of air, changing when the room ventilation was turned off, modified the temperature of the lamp envelope. The ventilation was turned off in the evening and the temperature of the envelope increased after the first two curves were taken. However the definite conclusion that we can draw is that the form of the band is defined by a single quartz reflection, further reflections merely "trim" the edge of the residual-ray band.

These spectra were all determined with a 1.14 mm CaF_2 filter inserted in the filter holder. The function of this filter, when the residual-ray apparatus is used as a pyrometer, is to remove the 20μ quartz residual-ray band.

The manner in which the residual-ray band is attenuated by successive quartz reflections is better established by a different experiment in which one successively focuses the telescope first on a blackbody at the steam point and then on one at the ice point. The results of these tests with 2, 3, 4, and 5 quartz reflections are given in Table I. The deflections listed there in the second main column were obtained using a 19.1Ω type HS Leeds and Northrup galvanometer connected to the thermopile. The readings were made at 1 m scale distance. The entrance aperture A was, in these tests, a circular hole 5 mm in diameter. The focal length of the pyrometer telescope was 250 mm. Thus, the divergence of the beam employed by the pyrometer is 1.1°. The deflections obtained for two quartz reflections, with a 24Ω portable galvanometer listed at $5.6\mu V$/division, was 22 divisions and, for five reflections, the deflection was 12 divisions.

These tests show that the reflection of a quartz crystal for the four crystal residual rays is 87 percent, the reflectivity of a quartz crystal for the three crystal residual rays is 84 percent and the reflectivity for the two crystal residual rays is 78 percent. These values are highly reproducible.

The cause of the discrepancy between these ratios and the ratios of areas under corresponding curves in Fig. 5 is to be laid partly to inequality of temperature between the collimating system and telescope system and partly to changes in the temperature of the lamp envelope.

USE OF THE RESIDUAL-RAY APPARATUS AND ATTACHED TELESCOPE AS A PYROMETER

Because all objects emit 8.8μ radiations at "room temperature" the procedure adapted in using the residual-ray apparatus as a pyrometer is to point its telescope first on a blackbody at the temperature of melting ice, then on a blackbody at the steam point and finally on the object whose temperature is desired. In each of these positions a galvanometer reading is made. *No shutter is used.*

To facilitate its use the apparatus is fitted with a finder and mounted on a wooden equatorial type telescope mounting. The galvanometer used with the instrument is mounted in a brass housing fitted with a mica observing window. The brass housing is, in turn, mounted

[11] The transmission limit of β-MgO in the infra-red falls between the transmission limits of quartz and fluorite. See J. Strong and R. T. Brice, Phys. Rev. **25**, 209 (1935). This material is accordingly suited for use as a so-called Rubens shutter. This shutter makes possible the use of a grating in the spectral region 8–16μ without a fore prism monochromator. For a description of the Rubens shutter see reference 10. The same piece of β-MgO studied by Strong and Brice has been tested with a prism spectrometer by Dr. A. Adel who gets the following transmissions (for a 1-cm layer):

λ	4μ	5μ	6μ	7μ	8μ
T	0.87	0.88	0.69	0.23	0.01

on a telescope tripod. This galvanometer has been used in the field without any difficulty.

The ice blackbody is a cork-insulated wide-mouth jug containing melting cracked ice. The other blackbody is made from an ordinary square five-gallon oil can. In one of the sides of the oil can near the top there is a re-entrant conical cavity. Both the can and the cone are made of tin plate. This re-entrant cone has its tin removed and is coated with a light film of rust to make it emit more nearly as a blackbody. A layer of water in the bottom of this can is boiled so that the cone comes to the boiling temperature. A condenser returns the water boiled out of the can.

The observations with the pyrometer consist of the readings of the galvanometer when the telescope is focused on the ice blackbody, d_0, when it is focused on the steam blackbody, d_{100} and when it is focused on the object in question, d_t.

New Temperature Scale

This pyrometer uses the intensity of emission of radiation in the 8.8μ quartz residual-ray band as the thermometric property. For convenience a new temperature scale and degree is employed. The degree is designated °R. 1°R causes a deflection of the galvanometer from its ice point position equal in magnitude to 1/100 of the deflection produced by the conical blackbody at 100°C. Thus, the temperature on the new scale is defined by the equation

$$t_{\circ R}=100(d_t-d_0)/(d_{100}-d_0).$$

It is noteworthy that the radiation temperature °R of a body is directly proportional to the net flux of radiation (at 8.8μ) between that body and a blackbody at 0°R. The difference between this flux and that of the surroundings is representative of heat transfer by radiation from the body to its surroundings.

The differences between temperature as defined by the new scale and the ordinary centigrade scale are given in Tables II and III. Accordingly, $t_{\circ R}$ are changed to $t_{\circ C}$ by adding or subtracting an appropriate $\Delta t_{\circ R}$ to $t_{\circ R}$. The value of $\Delta t_{\circ R}$ for any given temperature $t_{\circ R}$ can be taken from the tables or interpolated from values given there. Differences between the new temperature scale and the centigrade scale are determined by the wave-length in the residual-ray band. The data of Tables II and III are computed using Planck's law and taking the residual-ray band to be a strictly monochromatic band of radiation at 8.8μ.

Calibration

Calibration observations in the temperature interval between 0° and 50°C were obtained using the emission of a third blackbody filled

Table II.

°C	°R	Δ°R
0	0	0
5	2.8	2.2
10	5.9	4.1
15	9.2	5.8
20	12.6	7.4
25	16.3	8.7
30	20.2	9.8
35	24.4	10.6
40	28.8	11.2
45	33.4	11.6
50	38.2	11.8
55	43.4	11.6
60	48.6	11.4
65	54.2	10.7
70	60.0	10.0
75	66.0	9.0
80	72.4	7.6
85	79.0	6.0
90	85.8	4.2
95	92.8	2.2
100	100.0	0

Table III.

−°C	−°R	−Δ°R
0	0	0
5	2.6	2.4
10	5.1	4.9
15	7.4	7.6
20	9.4	10.6
25	11.3	13.7
30	13.1	16.9
35	14.7	20.3
40	16.1	23.9
45	17.4	27.6
50	18.5	31.5
55	19.5	35.5
60	20.4	39.6
65	21.2	43.8
70	21.9	48.1
75	22.5	52.5
80	23.0	57.0
85	23.4	61.6
90	23.8	66.2
95	24.1	70.9
100	24.3	75.7
273	25.12	—

with oil and heated with a flame. The temperature of this blackbody was measured by a mercury thermometer. Agreement was obtained between $t_{\circ R}$ as observed and calculated to $\pm 0.1°R$. The discrepancies between $t_{\circ R}$ as observed and calculated might have been less if a more refined experiment had been conducted. However, since the discrepancies are of the same magnitude as the observational uncertainties in the field they leave no doubt as to the legitimacy of using the calculated results given in Table II.

Table III has been checked at the liquid oxygen boiling point. The temperature of liquid oxygen on the basis of an 8.8μ isochromatic curve is $-25.12°R$ whereas on the basis of an 8.9μ isochromatic it is $-25.66°R$. Observations were made on liquid oxygen with the instrument mounted on the roof of the laboratory. They were made continuously over a forty-eight-hour period during which the instrument was exposed to sunshine during the day and sky radiations and cold air at night. These observations exhibited a spread of temperatures ranging from $-25.2°R$ during the night to $-26.4°R$ during daytime. The best values taken around midnight, gave as the probable value of the liquid-air temperature $-25.5°R$. Thus, the appropriate isochromatic seems to depend somewhat on the temperatures being measured. At low temperatures a value of $\lambda = 8.83\mu$ is probably best, while high temperatures will require an isochromatic for a wave-length slightly shorter than 8.8μ. The error introduced by assuming the residual-ray band to be narrow is a minimum at 54.5°C because the maximum emission of a blackbody occurs at 8.8μ at this temperature. In this connection one should compare the energy distribution given in Fig. 7 for source at 1000°C with the bottom curve of Fig. 5 for a 100°C source. It is intended in the future to determine the data presented in Tables II and III empirically.

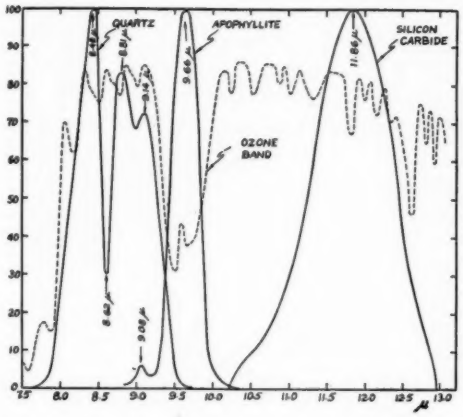

FIG. 7. Residual-ray bands obtained with five reflections of quartz, of apophyllite and of carborundum. The absorption of the atmosphere in the zenith direction for approximately 1.5 cm precipitable water. (This curve is determined in part by measurements using apparatus shown in Fig. 4 and observing the intensity of sunlight in the quartz and carborundum residual-ray bands. But the details are derived from A. Adel's curve, Astrophys. J. 89 (1939).)

TABLE IV.

°C	$\left(\frac{\Delta°C}{\Delta°R}\right)$
+100	0.7
+ 80	0.8
+ 60	0.9
+ 40	1.1
+ 20	1.4
0	1.8
− 20	2.5
− 40	3.7
− 60	5.9
− 80	11
−100	25

SENSITIVITY

The microvolt sensitivity of the pyrometer is $0.67\mu V/°R$ (using a beam having 1.1° divergence). The resistance of the thermocouple is 25Ω. Using the L and N type HS galvanometer it is not difficult to make a routine temperature measurement in two minutes accurate to 0.1°R. Each reading takes about $\frac{1}{2}$ minute.

The change in centigrade temperatures necessary to produce a change of 1°R is given in Table IV. It will be seen that the accuracy of the pyrometer decreases at very low temperatures. At −100°C a change of temperature of 25°C is required to produce a deflection corresponding to 1°R.

The new instrument is not as sensitive for high temperatures as the ordinary optical pyrometer. The virtue of the instrument as applied to high temperature measurement, as for example, the temperature of the sun, lies in the physical information it yields. Preliminary

measures indicate an "infra-red temperature" for the sun of 7080°K which is greater than the ordinary accepted temperature of 6000°K.

Transparency of the Atmosphere

Figure 7 shows the distribution of the energy in the quartz residual-ray band. Fig. 7 shows the energy distribution in the residual rays of the mineral apophyllite and in the residual rays of carborundum crystals. The transmission of the atmosphere is also given in Fig. 7 for about 1.5 cm water vapor content (precipitable water). It is to be noted that the wave-lengths of the quartz residual rays fall in a spectral region characterized by a high transmission. The entire optical path of the atmosphere absorbs only some 25 percent of the quartz residual rays.

Because of its meteorological significance, the transmission for the whole atmosphere is being studied with the three residual rays shown in Fig. 7. Although, as a result of this study, there may be a minor revision of the values indicated for the transmission of the atmosphere we can take them as qualitatively correct and conclude that for all ordinary terrestrial work where optical paths of but a few hundred yards are involved the effect of the atmosphere is inconsiderable.

Emission of Various Bodies

The residual-ray pyrometer described here is suited for two types of work. In the first comes the observations on objects for which not the temperature but rather the infra-red emission and radiative heat transfer are of primary importance. The walls of a building (see Fig. 8), the ground, leaves, grass and so forth, are examples. For such objects it is not necessary to know the emissivity. The effective radiation temperature, °R, is representative of the flux of energy emitted by the object.

Observations on bodies where one is interested primarily in temperature constitute the second type of work. Here it is necessary to know the emissivity of the body and make a correction for it. Also, it is necessary to use Tables II or III and convert °R to °C.

The human skin has been proved to radiate as a blackbody to within 2 percent.[1] Leaves, grass and earth are undoubtedly quite black at 8.8μ. Notable for its low emissivity is an aluminized plate of glass. When the aluminized side of such a plate heated to about 200°C is exposed to the pyrometer, the galvanometer scarcely swings. But, if the plate is turned over and the uncoated glass surface is exposed, the galvanometer swings quickly off scale.

An interesting experiment with the pyrometer is to focus its telescope on a lighted incandescent lamp. As one would expect the temperature indication is not immediately effected by turning off the filament. This shows that the instrument does not "see" the tungsten filament because the glass envelope is opaque at 8.8μ. The instrument "sees" only the glass envelope and this cools off very slowly just after the filament is turned off, later the cooling of the envelope is more rapid.

Another experiment, to determine the response of the instrument to the radiations not in the 8.8μ band, consisted in focusing the telescope on the sun and interposing a thin glass plate in the optical path. A clear glass plate transmits a large fraction of the energy in sunlight. When it was interposed in the optical path the galvanometer returned to within 1 percent of its zero reading.

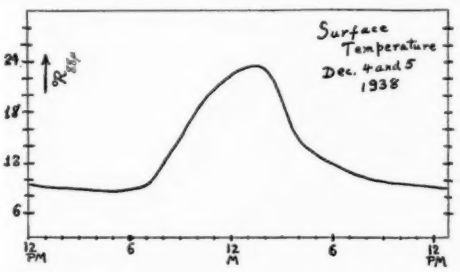

Fig. 8. Diurnal variation of the surface temperature of the south wall of the Mudd Geology Building (exposed to sunlight).

Thus, 99 percent of the response of the instrument to sunlight is due to energy in the 8.8μ band. It is expected, for colder sources, which have relatively more 8.8μ energy, that the response of the instrument is entirely confined to the 8.8μ band.

Modifications of the Pyrometer

There are many ways in which the new residual-ray pyrometer can be modified. First,

we see from Fig. 5 that the residual-ray band of quartz is already defined and well isolated by two reflections. One can make an instrument with two, three, four or five quartz reflections. The smaller number is naturally best suited for the measurement of temperatures near to the temperature of the instrument. Thus, one could make a simple but effective two-crystal residual-ray pyrometer for determining temperatures between -25 and $+50$°R (especially if the maximum polarization were obtained by slightly increasing the angles of incidence on the quartz plates over that indicated in Fig. 4).

There are various ways of assembling the two- and three-crystal apparatus. One which is being tested uses a rocksalt lens as telescope. A plane parallel quartz plate, mounted diagonally halfway between the lens and its focus, removes the 8.8μ band but lets the visible light pass on to an eyepiece. Thus is formed the finder. The residual-ray band is further isolated with a two-quartz spherical-mirror combination arranged with its polarizing azimuth crossed in respect to the polarizing azimuth of the first (diagonal) quartz plate. The elliptical mirror and thermopile are mounted in an appendage which is protected with a Dewar bottle.

It is necessary to handle this apparatus with care to preserve the optical surface on the deliquescent rocksalt lens. Where it is impossible properly to care for an optical surface on a rocksalt lens it is preferable to use a mirror telescope. A spherical mirror can be employed "off axis" if the instrument is to be used only for observing large surfaces. If the instrument is to be used on bodies of small angular diameter, a special off-axis telescope is necessary.

The thermopile housing of the present apparatus is closed with a KI window. This gives the instrument the flexibility desired in an experimental model. Because the KI window is transparent to wave-lengths as long as 33μ it is possible with the apparatus shown in Fig. 4 to study the residual-ray bands of many crystals. However, the use of a KI window requires a drying agent. To avoid the use of a drying agent in an instrument where flexibility as regards wave-length is not needed the thermopile housing may be closed with a nondeliquescent CaF_2 window. This window also affords the necessary filtering of the 20.75μ quartz residual-ray band.

Finally, some residual-ray band other than the quartz residual-ray band may be suited for certain investigations. For example, the apophyllite residual-ray band at 9.6μ is used for determining the ozone temperatures. The calcite residual-ray band can be used for making humidity measurements.

Absolute Humidity Measurements

The wave-lengths in the calcite residual-ray band (centered at $\lambda = 6.7\mu$) absorb water vapor strongly. For the wave-length band obtained with the two-crystal residual-rays (calcite crystals cut perpendicular to optical axis and mounted on turrets R_3 and R_4), the fractional absorption, A, is related to the water vapor in the optical path, τ, as follows

$$A = 4.4\sqrt{\tau}.$$

Here τ is expressed in centimeters of precipitable water. This result was obtained under atmospheric conditions using a blackbody radiation source at approximately 80°C. The water vapor concentration in the optical path was 13×10^{-6} g per cc. The air path used ranged from 135 to 2406 cm. The above relation holds for a water vapor path up to only 0.02 cm precipitable water.

The Koremetron: An Instrument for Measuring Pupillary Diameters

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(Received September 29, 1939)

The theory and construction of an apparatus which permits the rapid measurement of pupillary diameters is presented. This instrument, the koremetron, is portable, and uses the same principle as that involved in observing entoptical phenomena.

CHANGES in pupillary diameter are widely used in diagnosis, physiology and psychology as an index of nervous function. Quantification of these changes by current methods—principally photography—is tedious and difficult. In cooperation with the Department of Psychology of Harvard University, the writer has designed and constructed a portable apparatus which permits a rapid measurement of the pupillary diameter of one's own eye. It is the purpose of this communication to discuss the theory and construction of this instrument, which has been termed the koremetron.

Theory

It has been known for a long time that the view obtained by the eye of a point source of light which is out of focus permits one to see certain aspects of the structure of the eye—the so-called entoptical phenomena.[1] So far as the writer knows, however, this arrangement has not been used to afford a measure of the pupillary diameter, although it is admirably adapted to do so.

It will be shown that, other things being equal, the diameter of the disk of light which is seen as the out-of-focus image of the source is exactly proportional to the diameter of the pupil of the eye. In order to make clear the physical situation, we shall first treat a simplified example, and then proceed to the general case.

We shall assume for the moment that the eye is *perfect*, by which we mean that it is focused on infinity; we assume further that the point source of light is at infinity, and that the image on the retina is put out of focus by means of a short focus lens of focal length f placed x centimeters in front of the eye. A transparent scale is placed in front of the lens at the focal point. Fig. 1a represents the situation. The parallel light from the source is converted by the lens into a cone of light converging on a point inside the observer's head which is f centimeters behind the lens. At the iris, however, the cone is reduced to a cone of length $f-x$ with a base of diameter d, where d is the diameter to be measured. Thus the round image is derived from a cone of light with an angular spread given by

$$\theta \leqq 2 \tan \tfrac{1}{2}\theta = d/(f-x). \qquad (1)$$

The transparent scale placed at the focus of the lens will be seen clearly, and the diameter of the circle of light as read on the scale we shall call s. Then the angle which corresponds to the length s is

$$\theta \leqq 2 \tan \tfrac{1}{2}\theta = s/f. \qquad (2)$$

Combining (1) and (2), we have

$$d = s(1-x/f). \qquad (3)$$

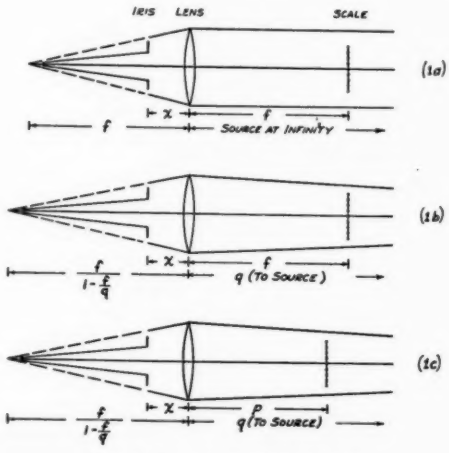

FIG. 1. For explanation see text

[1] Helmholtz, *Physiological Optics* (Optical Society of America, 1924), Vol. 1, p. 204.

FIG. 2. The koremetron with the battery box.

We see in this case that if x is negligibly small compared with f, the scale reads the pupillary diameter directly.

If now the point source is not infinitely distant, but is q centimeters in front of the lens as in Fig. 1b, the converging cone will no longer be f centimeters long, but will have a length of $fq/(q-f)$ centimeters. We have now

$$\theta \leqq 2 \tan \tfrac{1}{2}\theta = d/(fq/(q-f)-x) \qquad (4)$$

and with (2), this leads to

$$d = s(q/(q-f)-x/f). \qquad (5)$$

If the eye may not be considered to be focused on infinity, the relations become more complicated. Let the eye require a correction of N diopters to give perfect vision—that is to say, a lens of focal length $100/N$ centimeters would have to be put close before the eye in order that it focus on infinity when unaccommodated. Then the eye, relative to a perfect eye, has itself a focal length of $F = -100/N$ centimeters. It is now easy to show that the scale should be put a distance in front of the lens given by

$$p = f(F-x)/(F+f-x) \qquad (6)$$

and a little further use of the lens equations yields

$$\theta \leqq 2 \tan \tfrac{1}{2}\theta = sf/F(f-p). \qquad (7)$$

Eq. (4) is still valid, since it involves neither the correction of the eye, nor the scale distance. The result of eliminating θ and p among (4), (6), and (7) is

$$d = s\left[\frac{fq-x(q-f)}{q-f} \cdot \frac{f+F-x}{fF}\right]. \qquad (8)$$

The bracket is the ratio between the diameter of the pupil, and the apparent diameter as read on the scale, for the general case. It is somewhat cumbersome, but it need be calculated only once for a given optical system and a given observer.

If the eye is astigmatic, the image will not be round, but elliptical, in which case it is suggested that the scale be oriented so that the longest diameter is measured; then the value of N to be used in finding F is the algebraically smallest value which can be obtained from the combination of spherical and cylindrical corrections.

The diameter measured by the koremetron is, of course, not the anatomical diameter, but is practically identical with the diameter as measured on a photograph of the eye. That this is true follows from the fact that we have not traced any rays into the eye, but have simply considered the iris as located at the cornea. Very little error is introduced by this procedure.

DESIGN

The general construction of the instrument may be seen from the photographs. The pinholes, illuminated by flashlight lamps, are at the far end of the tubes in Fig. 2.

It was decided to build the apparatus in duplicate, with a lens and pinhole for each eye, in order to hold fast the convergence of the eyes. Convergence of the eyes would bring with it a certain amount of contraction of the pupils. The use of two tubes also makes it possible to control the general illumination of the field of each eye; there is another lamp in each tube to control the interior illumination. These lamps are halfway down the tubes, and are shielded from the lenses so that they illuminate only one end of the tube. Only one of the tubes contains a scale.

There is no provision to change the interocular distance; this is fixed at 6.40 centimeters.

The brass tubes are 30 centimeters long with an outside diameter of 2.5 inches. The scale is mounted in a short tube which slides snugly inside one of the long tubes. The two long tubes are exchangeable so that the pupillary diameter of either eye may be measured. The short tube slides longitudinally inside the long tube to vary the distance p from the scale to the lens, while the short and long tube rotate together to orient the scale so that it can measure the major axis of an elliptical image, no matter how this axis

may happen to be inclined. The longitudinal and rotational motions may be clamped independently.

It was found on the optical bench that the simple device of a transparent scale with opaque rulings was unsatisfactory because although it permitted a clear measurement of the diameter of the disk of light, it failed completely to provide a plane for the eye to focus upon: the rulings formed a sharp shadow on the retina of the light from the pinhole whether the scale was at the proper distance or not. Now it is important for two reasons that the eye be focused steadily upon the focal plane of the scale: (1) a different focus of the eye will alter the optical system slightly; (2) more important is the fact that if the accommodation of the eye is not fixed, but is permitted to fluctuate, spurious

FIG. 3. Showing the scale by which is measured the apparent diameter of the disk of light.

contractions will be produced when the eye accommodates. A different type of scale, which is shown in Fig. 3, is therefore used in the apparatus. Ten parallel slits, each one centimeter long, 0.7 millimeter wide, and one millimeter apart were cut in a thin sheet of brass with an engraving machine. Rulings one millimeter apart were then cut transverse to the slits. The rulings are illuminated by oblique light so that they stand out as dashed white lines against a black background, with the round spot of light showing through the slits.

It is essential that the distance from the scale to the lens, which we have called p, be capable of accurate adjustment. This is afforded by a narrow steel scale set flush with the surface of the inner tube. The square opening through which the scale is read may be seen clearly in Fig. 2. It is not necessary to know F precisely, for if the instrument is adjusted for an F slightly

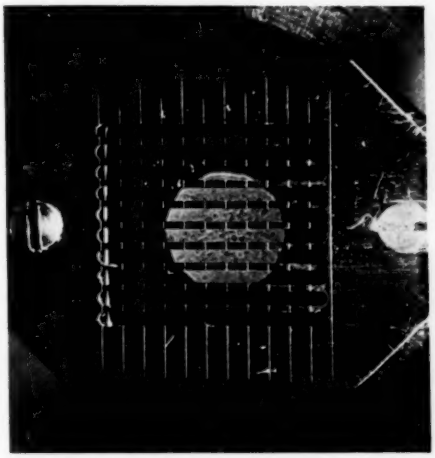

FIG. 4. Showing what is seen by the eye. The picture was taken by putting the scale at the focus of the lens ($p=7.5$ cm); the camera was focused upon infinity and placed in front of the eyepiece with a ¼-inch stop between the eyepiece and the camera lens.

smaller (algebraically) than the precise value, a slight accommodation of the eye will take care of the difference. It is least tiring to the eye, however, if p is made as large as possible with the scale still in focus. If one wishes, one may adjust the instrument in this manner, and then calculate F from p:

$$F=x-fp/(p-f). \qquad (6')$$

Once the length of the long tubes had been decided upon, the optimum focal length of the lens was determined by the angular field which the eye could comprehend at once, since while estimating the diameter s on the scale, the eye must look steadily at the center of the spot because a movement of the eye would cause a change in the position of the spot relative to the scale. The lenses used in the present instru-

ment have a focal length of 7.5 centimeters, and the value of p may be varied from 4.5 to 11 centimeters, so that eyes requiring rather large corrections may be handled without difficulty.

The battery box contains two No. 6 dry cells, and has a rheostat for each of the five lamps: two for the pinholes, two for the interior illumination, and one for the scale.

ACKNOWLEDGMENT

The writer wishes to express his appreciation to Professor F. E. Nurse of the Department of Philosophy at the University of Toledo for suggesting the name of the instrument, and to Dr. J. G. Beebe-Center of the Department of Psychology at Harvard University for many helpful suggestions.

A Push-Pull Photoelectric Photodensitometer for Determining Fine Structure in Ultraviolet Absorption Spectra

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(Received September 28, 1939)

The photoelectric photodensitometer described was developed for determining fine structure from "match-point" spectrum photographs. It consists of a push-pull photo-cell circuit, transformer coupled to an amplifier and cathode-ray oscillograph, and two optical systems with a.c. operated light sources for projecting the spectra onto the photo-cell slit and for forming an image of the wave-length scale which indicates the region of the spectrum being matched. As the projected spectra are moved horizontally across the slit, a change in phase occurs in the a.c. output of the photo-cell circuit on passing through a match point. The corresponding change in oscillograph pattern indicates whether the absorption is increasing or decreasing, and permits true match points to be distinguished from decreases in intensity due to dense spectrum lines in matching line spectra.

IN 1931, Heyroth and the author[1] published absorption spectra of several organic compounds related to nucleic acids in which, in addition to the broad maxima at or near 2600A characteristic of these compounds, many smaller subsidiary peaks were shown. These fine structure peaks were too small to be detected by the usual methods of ultraviolet absorption spectrophotometry (such as the sector photometer) then in use. We were enabled to find them because the method we employed[2] involved certain precautions which resulted in greater accuracy in the region of high extinction coefficients. Nevertheless, it was sometimes difficult to reproduce the subsidiary peaks because they approached the limit of accuracy of the method.

More recently, several investigators[3] have reported fine structure in the bands of various amino acids, proteins, etc. Some of them worked at low temperatures, following the methods of Kronenberger,[4] Conant and Crawford,[5] Arnold and Kistiakowsky,[6] and others. In this later work, however, location of the fine structure has depended upon visual observation of the spectrographic plates or examination of photodensitometer traces for irregularities, and it has not been possible to evaluate the fine structure peaks quantitatively.

The push-pull photoelectric photodensitometer about to be described was developed in order to permit the objective determination and quantitative evaluation of the fine structure from match point spectrograms of the ultraviolet absorption spectra of organic compounds. The need for such a method arises not only from the fact that the fine structure affords more complete empirical data for the identification of such compounds, but also from the relation of the fine structure peaks to the vibration frequencies of the individual molecular groupings—a relationship which permits quantitative expressions to be worked out for the influence of substituent groups, etc., once the fine structure is established.[7]

The new photodensitometer has been used to locate the match points on spectrum plates photographed with a Hilger quartz spectrograph and Spekker photometer. The spectra to be matched are projected through a slit onto the plates of an RCA type 920 double photo-cell, a narrow band of the absorption spectrum falling on one photo-cell plate and the corresponding portion of the comparison spectrum falling on the other. A 100-watt 16-mm motion picture

[1] F. F. Heyroth and J. R. Loofbourow, J. Am. Chem. Soc. **52**, 3441 (1931).

[2] J. R. Loofbourow, Bull. Basic Sci. Research **5**, 46 (1933).

[3] Lavin, Northrup and Taylor, J. Am. Chem. Soc. **55**, 3497 (1933); Ross, J. Biol. Chem. **104**, 531 (1934); Lavin and Northrup, J. Am. Chem. Soc. **57**, 874 (1935); Feraud, Dunn and Kaplan, J. Biol. Chem. **112**, 323 (1935); Coulter, Stone and Kabot, J. Gen. Physiol. **19**, 739 (1936); Holiday J. Sci. Inst. **14**, 166 (1937); Holiday, Biochem. J. **31**, 1299 (1937); Lavin and Stanley, J. Biol. Chem. **118**, 269 (1937).

[4] H. Kronenberger, Zeits. f. Physik **63**, 494 (1930).

[5] J. B. Conant and F. H. Crawford, Proc. Nat. Acad. Sci. **16**, 552 (1930).

[6] L. B. Arnold and G. B. Kistiakowsky, J. Am. Chem. Soc. **54**, 1713 (1932).

[7] J. R. Loofbourow, *Studies Inst. Divi Thomae*, in publication.

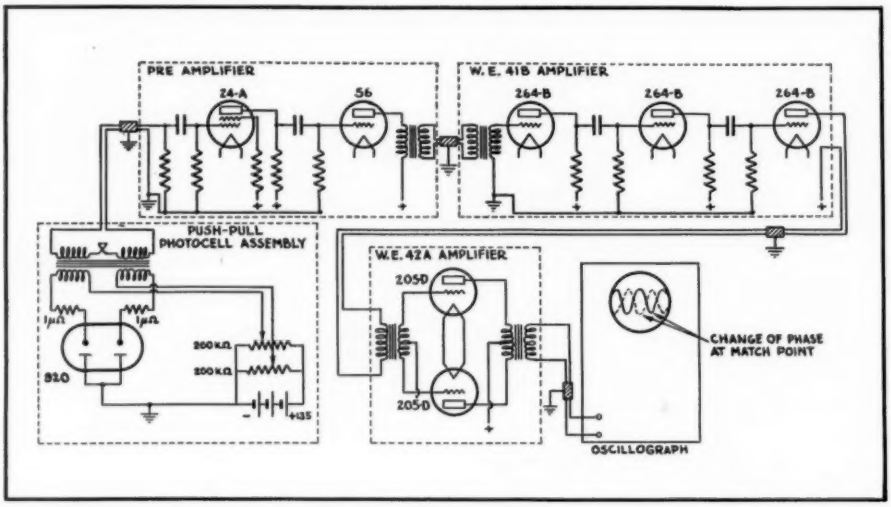

FIG. 1. Schematic diagram of push-pull photoelectric photodensitometer (power supply circuits omitted).

projector lamp operated from a 100-v 60-cycle a.c. line is used as the light source. The 60-cycle flicker of the lamp intensity is sufficient to provide alternating impulses in the photo-cell circuit, without the necessity of "chopping" the light beam. A high pressure mercury arc of similar wattage has also been employed, and has been found to give somewhat greater sensitivity.

The positive terminal of a 135-volt photo-cell battery is connected to the primary mid-tap of a special high impedance push-pull transformer[8] the end terminals of which are connected through one-megohm resistors to the photo-cell anodes, the cathodes and negative battery terminal being grounded. Thus, impulses in the two half-cells are impressed on the transformer 180° out of phase, tending to cancel each other. Potentiometers in the anode circuits provide means of adjustment for null output with a uniformly illuminated slit.

The push-pull transformer output is amplified by a two-stage, resistance-capacity coupled amplifier of conventional design, using one 24A and one 56 type tube. This is followed by a Western Electric 41B amplifier, consisting of three resistance-capacity coupled stages employing 264B tubes, and finally by a Western Electric 42A amplifier, which is a single, transformer-coupled push-pull stage using 205D tubes. Battery operation is employed throughout up to the output stage, which is a.c. operated. The total gain of the three amplifiers is 121 db at 1000 cycles and 112 db at 60 cycles.

The output of the 42A amplifier is connected to the vertical deflection plates of an RCA 3-inch oscillograph. This has a single stage amplifier with a maximum gain of 32 db. Hence the total available gain is 144 db at 60 cycles (100 db has been found adequate for most measurements).

The plate holder and optical assembly are mounted on the carriage of a lathe bed, the cross feed providing means of moving the horizontal spectra across the vertical photo-cell slit, and a milling attachment fastened to the cross feed serving to move the plates vertically until the desired pair of spectra is projected onto the slit. A wave-length scale attached to the cross feed is projected by a separate optical assembly onto a reference screen near the photo-cell head, so that the wave-length corresponding to the center of the image on the slit may be read directly.

[8] This transformer, which is of the type used in motion picture push-pull soundheads, was kindly loaned by Mr. M. G. Thomas of the Altec Corporation.

To match spectra, the horizontal oscillograph sweep is adjusted for a 60-cycle standing wave pattern, the photo-cell potentiometers set for null oscillograph deflection with uniform slit illumination, and the pair of spectra to be matched is (after proper vertical positioning) racked horizontally across the photo-cell slit. Passage through a match point is indicated by a null reading, accompanied by a change in oscillographic pattern indicative of a 180° phase shift. The direction of phase change indicates whether the absorption is increasing or decreasing, for opposite phases correspond to greater light intensity in the absorption spectrum image on the one hand and the comparison spectrum image on the other.

Many photoelectric densitometers have been described in the literature.[9] This one differs principally in that the use of a double photo-cell simplifies the mechanical and optical set up for a null-method matching of adjacent spectra, and that the use of a push-pull circuit in combination with cathode-ray registration affords the advantage of a visually indicated phase shift at the match point. The latter facilitates the matching of line spectra by discriminating between low readings due to dense spectrum lines and true match points, and increases the accuracy with which the settings for null readings may be made. If a rectifier type output meter (which, of course, does not indicate the phase shift) is used instead of an oscillograph, settings are considerably less accurate and, in our experience, minor peaks are often overlooked.

TABLE I. *Example showing range of values obtained in repeated trials with photoelectric method for matching spectra. Material, uracil.*

EXTINCTION	WAVE-LENGTHS OF MATCH POINTS HIGHEST	LOWEST
0	2848	2848
0.1	2845	2845
0.2	2818	2815
0.3	2806	2806
0.4	2234	2230
	2343	2341
	2789	2788
0.5	2212	2210
	2376	2374
	2782	2772
0.6	2392	2391
	2755	2751
1.2	2500	2499
	2692	2690

[9] See G. R. Harrison, J. Opt. Soc. Am. **24**, 59 (1934) for a review of such devices.

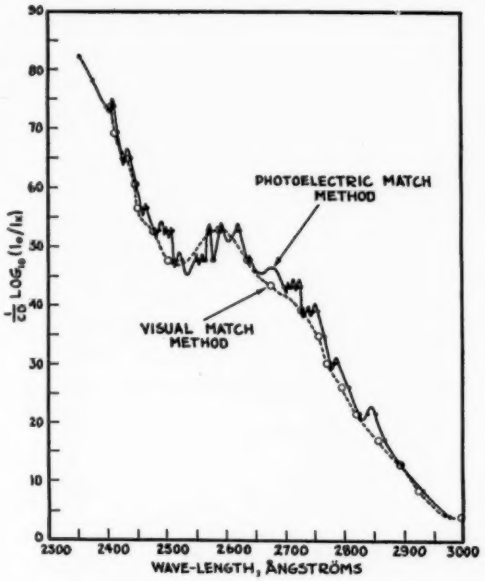

FIG. 2. Example of absorption spectrum showing increased resolution obtained with push-pull photoelectric matching device as compared with visual matchings.

A schematic circuit diagram of the device is shown in Fig. 1. Table I is an example of the reproducibility of readings, which is, in most instances, within the curve-plotting error. The sensitivity is sufficient to give consistent quantitative objective readings of fine structure peaks of the type reported for such compounds as uracil in our earlier papers.[1] Fig. 2 is a typical absorption curve determined with the device.

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